GEOPHYSICS, VOL. 45, NO. 5 (MAY 1980); P. 973–976, 1 FIG., 2 TABLES.

Short Note

Aeromagnetic survey design

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INTRODUCTION

Aeromagnetic surveys are flown with a wide variety of terrain clearances, sampling rates, and line spacings. The results are generally presented as contour maps, implying that the survey grid defines the continuous magnetic field sufficiently well to justify interpolation.

It is possible to investigate this assumption by considering the spatial spectrum likely to be encountered. The shortest spatial wavelength λ_N, which may be correctly identified by a regular line of samples of spacing Δx, may be termed the Nyquist wavelength λ_N given by

$$\lambda_N = 2\Delta x.$$

Any wavelength shorter than λ_N is aliased into a wavelength longer than λ_N (Blackman and Tukey, 1959). If the user is not to be seriously misled, the true spectrum of the magnetic field must contain no significant power at wavelengths shorter than the chosen λ_N. It is thus of importance to determine a priori the most appropriate value for Δx in terms of height of the sensor above source bodies.

EXPECTATION POWER SPECTRA

Total field survey

Spector and Grant (1970) showed that the expectation value of the power spectrum profile from an ensemble of magnetized blocks, when reduced to the north magnetic pole, is given by

$$\langle E(r)\rangle = 4\pi^2\bar{k}^2 \exp(-2\bar{h}r)\langle C^2(r)\rangle\langle S^2(r)\rangle, \quad r\Delta h < 0.5, \qquad (1)$$

where

r = the wavenumber,
$\bar{k}$ = the mean value of the magnetic moment per unit depth,
h = the elevation difference between the top surfaces of the magnetic blocks and the sensor,
Δh = the amplitude of variation of h,
$\bar{h}$ = the mean value of h,
$\langle C^2(r)\rangle$ = a term dependent on the depth extent of the sources, and
$\langle S^2(r)\rangle$ = a term dependent on the dimensions of the upper surfaces of the blocks.

The term $\langle C^2(r)\rangle$ approaches unity for sources of considerable depth extent, and the term $\langle S^2(r)\rangle$ approaches unity for sources of small upper surface area. Hence, for the simplified case of an assemblage of small sources of considerable depth extent, equation (1) reduces to

$$\langle E(r)\rangle = 4\pi^2\bar{k}^2 \exp(-2\bar{h}r). \qquad (2)$$

For a given survey spacing Δx, there will be a Nyquist wavenumber r_N given by

$$r_N = 2\pi/\lambda_N = \pi/\Delta x. \qquad (3)$$

Then the fraction of the power aliased F_T may be determined as

$$F_T = \left[\int_{r_N}^{\infty}\langle E(r)\rangle\,dr\right] \div \left[\int_0^{\infty}\langle E(r)\rangle\,dr\right]$$
$$= \exp(-2\pi\bar{h}/\Delta x). \qquad (4)$$

Here Δx should be taken to be the line spacing or

Manuscript received by the Editor February 1, 1979; revised manuscript received August 27, 1979.
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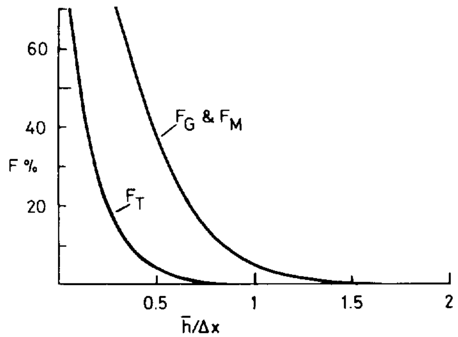

FIG. 1. The expected percentage of aliased power F from an aeromagnetic survey, conducted at mean clearance $\bar{h}$ and sample and line spacing Δx, versus the dimensionless parameter $\bar{h}/\Delta x$. F_T = total field survey. F_G = vertical gradient (gradiometer) survey. F_M = total field survey over isolated point dipole.

in-line sample spacing, whichever is larger. The percentage of power aliased, as predicted by equation (4), is plotted against the dimensionless parameter $\bar{h}/\Delta x$ in Figure 1 and given at suitable line spacings in Table 1.

The requirement $r\Delta h < 0.5$, specified for the validity of equation (1), restricts the permissible variation in h. If $r\Delta h$ exceeds this limit, a sinh (hyperbolic) factor is introduced into equation (1) which has the effect of increasing the high-wavenumber power so that the estimate expressed in equation (4) is an underestimate of the expected aliasing. The terms $\langle C^2(r)\rangle$ and $\langle S^2(r)\rangle$ are monotonically opposing functions of r, both with asymptotic values of unity; in general, their effects on F_T tend to cancel. Thus, the fraction expressed by equation (4) may be taken as representative of the expected aliasing, provided h does not vary widely.

Table 1. Aliased power.

$\bar{h}/\Delta x$	F_T	F_G F_M
	Percent	Percent
0.25	21	79
0.5	4.3	39
1	0.19	5
2	3×10^{-4}	.03
4	—	4×10^{-7}

$\bar{h}$ = mean height of sensor above magnetic sources.
Δx = sample or flight line spacing.
F_T, F_G, and F_M = aliased power fraction expected from surveys of total field, vertical gradient, and total field over point dipole, respectively.

Gradiometer survey

The expectation power spectrum resulting from a gradiometer (vertical gradient) survey or from operating on the results of a total field survey using a gradient filter may be deduced by consideration of the spectrum of an equivalent layer of dipolar magnetization in the origin plane. This is given by Gunn (1975) as

$$M_f(u, v, h) = 2\pi(jLu + jMv - Nr) \cdot \cdot (jlu + jmv - nr) \cdot \cdot m_s(u, v)\,[\exp(-hr)]/r \qquad (5)$$

in the notation of this paper rather than Gunn's, and where

j = the complex operator,
L, M, N = the direction cosines of the direction of magnetization,
l, m, n = the direction cosines of the magnetic component measured,
u, v = the components of the horizontal wavenumber r, and
$m_s(u, v)$ = the wavenumber spectrum of the surface density distribution in the origin plane of the dipolar magnetization.

It is apparent that r, the horizontal wavenumber given by $(u^2 + v^2)^{1/2}$, is closely related to the vertical wavenumber jr. This is a consequence of the fact that magnetic fields in free space obey Laplace's equation.

The spectrum of the vertical gradient may now be deduced by differentiation with respect to h. This introduces an extra factor r into the spectrum and thus a factor r^2 into the right-hand side of equation (2) to give

$$\langle E(r)\rangle = 4\pi^2\bar{k}^2 r^2 \exp(-2\bar{h}r). \qquad (6)$$

Hence, the aliased power fraction may be deduced, by the same means as that used to obtain equation (4), to be

$$F_G = [2(\pi\bar{h}/\Delta x)^2 + 2\pi\bar{h}/\Delta x + 1] \cdot \cdot \exp(-2\pi\bar{h}/\Delta x). \qquad (7)$$

The variation of F_G with $\bar{h}/\Delta x$ over the pertinent range is given in Table 1 and in the graph of Figure 1.

Individual anomalies

It may be considered desirable to calculate models of individual total field anomalies. In this case, the power spectrum from an ensemble of magnetized blocks is of little interest. An upper limit on the wavenumbers of interest may be deduced by obtaining the spectrum of a point dipole at the earth's surface, the sharpest likely magnetic anomaly. An appropriate starting point is equation (5). The point dipole is a dipolar magnetization distribution which is equivalent to a two-dimensional (2-D) Dirac delta function with its uniform wavenumber spectrum, so that $m_s(u, v)$ may be regarded as constant. The expression may be evaluated quite easily for the case of a vertical point dipole at the magnetic pole, for which $L = M = l = m = 0$ and $N = n = 1$. The result is

$$M_f(u, v, h) = 2\pi m_s(u, v) r \exp(-hr),$$

and the power spectrum becomes

$$E(r) = 4\pi^2 m_s^2 r^2 \exp(-2hr), \qquad (8)$$

where m_s is taken to be constant. Hence, the proportion of aliased power F_M is calculated as before. The resulting expression is identical to that obtained for F_G in equation (7) so that the F_M and F_G columns of Table 1 and the curves in the graph of Figure 1 are common. Here the expression for the aliased power fraction is exact because it does not depend upon any assumptions about the nature of a statistical population.

SURVEY DESIGN

Total field survey

The parameter $\bar{h}$ is the mean height difference between the sensor and the upper surfaces of the magnetic sources. Where there is thick sedimentary cover, it represents terrain clearance plus sediment and/or water layer thickness. However, for surveys over exposed igneous basement, it is simply the terrain clearance. The tolerable aliased power naturally depends upon the purpose of the survey. However, if any interpolation such as contouring is to be performed, then the aliased power should not exceed, say, 5 percent, giving a minimum value of 0.5 for $\bar{h}/\Delta x$. Thus, neither the sample spacing nor the line spacing should exceed $2h$ unless conditions make it clear that a low proportion of short-wavelength power is to be expected. This would occur, for example, if the magnetic fabric of the countryside showed a pronounced strike and flight lines were arranged perpendicular to that strike. The restriction on flight line spacing could then be relaxed somewhat, at the expense of definition of short features or structures such as faults with other strikes.

The results of such a survey could not, in general, be used with any confidence to calculate derived maps involving high-pass filtering, such as residual, downward continued, or gradient maps. The aliased power in such maps would be intolerably high. Nor could such survey results be used in modeling individual anomalies because high-wavenumber detail, crucial in such an application, would be aliased.

Gradiometer survey

Table 1 shows that a gradiometer (vertical gradient) survey conducted at the above suggested flight line spacing ($2\bar{h}$) would be intolerably aliased. Thirty-nine percent of the total power appearing in unexpected lower wavenumbers would distort the map to the point of being unrecognizable and highly misleading. This is well illustrated by Hood et al (1979) who show the results of a gradiometer survey flown over a test area at a terrain clearance and flight line spacing of 150 m which was then contoured using all the data, alternate flight lines, and every fourth flight line. Only the first of the three maps can be considered meaningful. This is not surprising since the expected aliasings at flight line spacings of 150 m ($\bar{h}/\Delta x = 1$), 300 m ($\bar{h}/\Delta x = 0.5$), and 600 m ($\bar{h}/\Delta x = 0.25$) are 5, 39, and 79 percent, respectively (Table 1).

Thus, gradiometer surveys should be flown at a line spacing equal to $\bar{h}$. Similarly, total field surveys, whose results will be processed to produce residual or gradient maps, should be flown at this spacing. Downward continuation could safely be performed to 0.5 $\bar{h}$, but no lower without digital low-pass filtering before the continuation.

Individual anomalies

The modeling of individual anomalies is crucially dependent upon the high-wavenumber end of the spectrum because most of the information concerning the detailed shape of the source being modeled is concentrated in this region. The spectra discussed here are all tapered at the high-wavenumber end by the exponential decay term so that the frequency folding inherent in the aliasing phenomenon ensures that most of the aliased power appears at the high-wavenumber end, making it unreliable in the very region where reliability is essential. Thus, reliable modeling is possible only if there is no appreciable aliasing. Reference to Table 1 shows that this occurs for sample and flight line spacings less than or equal to one-half $\bar{h}$. Naudy (1971) used a sample spacing of one-quarter $\bar{h}$ in a discussion of automated modeling. His choice of sample density clearly exceeds the minimum criterion suggested above and, as may be expected, his results are reliable.

Table 2. Maximum flight line spacings.

Survey type	Intended use	Δx
Total field	Contour map	$2\bar{h}$
Total field	Computation of gradient, etc., maps	$\bar{h}$
Vertical gradient	Gradient contour map	$\bar{h}$
Total field	Modeling of single anomalies	$\bar{h}/2$

Δx = maximum flight line or sample spacing.
$\bar{h}$ = mean height of sensor above magnetic sources.

CONCLUSIONS

It has been shown that a priori calculation of the expected aliasing is a simple matter. Table 2 summarizes the maximum flight line spacings which should be used in various types of survey if misleading results are to be avoided. The criterion can occasionally be relaxed somewhat if the magnetic fabric of the countryside has a pronounced strike. Since the effect of aliasing is mainly at the high-wavenumber end of the spectrum, the situation can always be improved by low-pass filtering (preferably upward continuation), but it is better avoided by sensible choice of $\bar{h}$. In-line sample spacing could advantageously be half that suggested in Table 2 because collection of this extra information is possible at minimal extra cost.

The choice of flying height itself depends upon the purpose of the survey. Regional surveys might be conducted at a height of 500 or 1000 m with appropriate flight line spacing. Broad features would be defined and local features suppressed. High-resolution local surveys might be flown at 150 m or even lower, defining features with scales down to hundreds of meters. Closer definition could most easily be achieved with ground surveys for which the relations between line spacing and sensor height are equally valid.

The high-resolution aeromagnetic survey flown by the Geological Survey of Canada (1977) over map sheets 31*F*/*a*-*h* meets, but does not exceed, the design criteria suggested. Unfortunately, many other surveys flown recently do not possess this resolution and thus the shallow, short-wavelength anomalies may well be intolerably aliased.

ACKNOWLEDGMENTS

I would like to thank Dr. P. L. McFadden for valuable discussions and Dr. P. J. Hood and M. T. Holroyd and P. H. McGrath for permission to quote their paper in reprint form. I would also like to acknowledge helpful suggestions from several anonymous referees.

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