Short Note: Euler magnetic structural index of a thin bed fault

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INTRODUCTION

Euler deconvolution (Thompson, 1982; Reid et al, 1990) has come into wide use as an aid to interpreting profile or gridded magnetic survey data. It provides automatic estimates of source location and depth. In doing this, it uses a structural index (SI) to characterize families of source types. Typical values are shown in Table 1.

Thompson (1982) showed that Euler's equation could usefully be written in the form

$$(x - x_o)\, \partial T/\partial x + (y - y_o)\, \partial T/\partial y + (z - z_o)\, \partial T/\partial z = N(B - T), \qquad (1)$$

where (x_o, y_o, z_o) is the position of a magnetic source whose total field T is detected at (x, y, z). The total field has a regional value of B. N is the structural index.

The structural index can be interpreted as the exponent in a power law expressing the fall-off of field strength versus distance from source. For magnetic data, physically plausible SI values range from 0 to 3. Values less than zero imply a field strength that increases with distance from source (and is infinite at infinity). Values greater than three imply quadrupole or higher order multipole sources.

The case of the thin bed fault (sometimes called the "two-sided fault") is examined below. It is shown that an SI of 2 applies.

FAULTED THIN BED

High sensitivity, high resolution aeromagnetic surveys now image structure in sediments on a routine basis (e.g. Grauch et al., 2001). This offers the possibility of new source geometries, including faults affecting magnetic strata (such as shale beds).

Werner (1955) quotes and re-derives a result by Rössiger and Puzicha (1933) for the magnetic anomaly of a thin dipping sheet edge. A more accessible source is Blakely (1995, 248). In the notation of this paper (figure 1) this is

$$DT_s(x) = \{C_1(x - x_o) + C_2 d\}/\{(x - x_o)^2 + d^2\}, \qquad (2)$$

where

$C_1 = -D\,(a \cos j + b \sin j)$,

$C_2 = D\,(-a \sin j + b \cos j)$,

$D = 2C_m M Dx$

j is the angle of dip of the sheet below the horizontal,

$a = F_x M_x - F_z M_z$,

$b = F_x M_z + F_z M_x$,

F_x, F_z are the direction cosines of the ambient field in the x and z directions,

M_x, M_z are the direction cosines of Magnetization in the x and z directions,

M is the scalar value of the magnetization,

Dx is the sheet thickness, and

C_m is a constant depending on the magnetic units ($m_o/4p$ H/m in SI units).

We can construct a thin bed fault model by juxtaposing two such dipping sheets, one with dip j of 180° at depth d and one with dip j of 0° at depth d+Dd (Figure 2). Substitution of these values gives for the left half of the model (j = 180°);

$C_1 = a\ D$,

$C_2 = -\ b\ D$,

and for the right half of the model (j = 0°);

$C_1 = -\ a\ D$, and

$C_2 = b\ D$.

If we set the x and y origins at the fault top and add the effects of the left and right halves of the model using Equation 2 (remembering the right half is at depth d + Dd), we obtain

$$DT_s(x) = D\{[(a\ x - b\ d)/(x^2 + d^2)] - [(a\ x - b\ d - b\ Dd)/(x^2 + d^2 + 2d\ Dd + Dd^2)]\}. \quad (3)$$

Make the vertical throw of the fault small compared to its depth, i.e. $Dd << d$. Then we may neglect the terms $2d\ Dd$ and Dd^2 above, and simplify to

$$DT_s(x) = D\ b\ Dd\ /(x^2 + d^2). \quad (4).$$

This is the magnetic effect of a thin bed fault. It clearly shows a fall-off of magnetic effects as the square of distance from source, suggesting an SI of 2. To prove this, we recognise that d in this equation is the z coordinate. We also recall that we set the origin at the fault, so $x_o = z_o = 0$, and we are dealing with a model, so the background B is zero. Thus

$$\partial DT_s(x)\,/\partial x = -2\,x\,D\,\mathbf{b}\,Dd\,/(x^2 + z^2)^2,$$

$$\partial DT_s(x)\,/\partial z = -2\,z\,D\,\mathbf{b}\,Dd\,/(x^2 + z^2)^2,$$

and Equation (1) becomes

$$-2\,x^2\,D\,\mathbf{b}\,Dd\,/(x^2 + z^2)^2 \;+\; -2\,z^2\,D\,\mathbf{b}\,Dd\,/(x^2 + z^2)^2 \;= -\,N\,D\,\mathbf{b}\,Dd\,/(x^2 + z^2),$$

which reduces to

$$N = 2.$$

DISCUSSION

The above derivation shows that the SI for the magnetic field of a thin bed fault is 2. This has several interesting and useful consequences:

1. We might reasonably hope to see such thin bed fault effects every time a fault of small throw affects a sufficiently magnetic shale band. This is commonly expected in sedimentary situations, especially if we remember that shales are more prominent magnetically than most sediments. Their magnetic susceptibility can be as high as 18×10^{-3} SI (Telford et al., 1976, 121).

2. It has been widely assumed that the highest useful SI for geologically reasonable 2D features is unity (thin sheet edge). Values like 2 were thought to apply only to cultural features like pipelines, powerlines and fences, or to geological features with

limited horizontal extent like narrow kimberlite pipes. This is a useful counter-example, which should be widely observable. My own work on proprietary data suggests that such faults are observable.

3. Equation 4 contains significant information about the anomaly amplitude. *D* and *b* are concerned with magnetization, bed thickness, and the various angles. But the fault throw, *Dd,* also appears. So the fault anomaly very properly disappears when the throw is zero, and it will be small for small throws. The whole derivation breaks down and becomes non-linear if *Dd* is not $<<$ *d*.

4. There is a common fallacy, that "the SI for a fault is zero". Reid et al. (1990) showed that an SI of zero could be used for a contact (faulted or not) of infinite depth extent. A modified form of the Euler equation (Equation 1) was necessary. Faults of finite depth extent may be expected to yield higher SI values. This note shows that, depending on the geometry, the magnetic SI value for a fault anomaly can be as high as 2, for geologically reasonable geometries. This is equivalent to the SI expected from a line of dipoles.

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Table 1 Euler magnetic structural indices

Source type	**SI**
Sphere or compact body at a distance	3
Line source (pipeline, narrow kimberlite pipe)	2
Thin sheet edge (sill, dike)	1
Contact of considerable depth extent	0

FIGURE CAPTIONS

Fig. 1. Thin sheet model, depth d, dip j.

Fig. 2. Faulted thin bed, depth d, fault throw $Dd << d$.

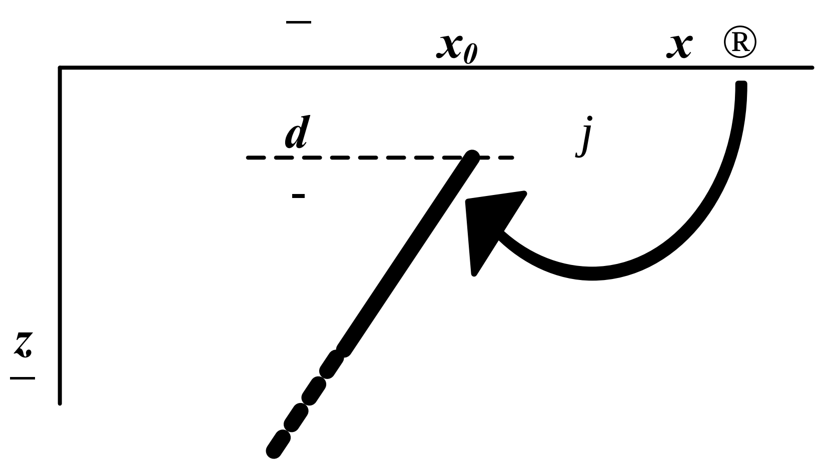

Figure 1. Thin sheet model, depth d, dip j.

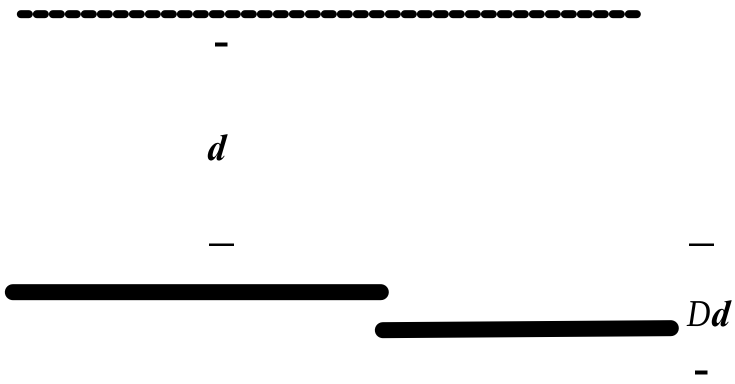

Figure 2. A faulted thin bed, depth d, fault throw Δd << d