

## The structural index in gravity and magnetic interpretation: Errors, uses, and abuses

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### ABSTRACT

The structural index (SI) is based on the concept of Euler homogeneity, a description of scaling behavior. It has found wide use in potential-field depth estimation and is a constant integer for simple sources with single singularities (points, lines, thin-bed faults, sheet edges, infinite contacts). For these cases, the SI is identical to the index of a simple power-law field fall-off with distance. The simple Euler formulation is only strictly correct for such simple sources and integer SI values. The widespread use of the simple Euler method on more complex structures, using fractional SI values is likely to produce misleading results because the SI is no longer a constant for any given source. We examine a recently published example that

used an arbitrary SI to estimate depth to the base of the crust for Africa and produced misleading results. Extension to more complex sources such as tabular bodies or thick steps requires one of several more generalized approaches, which recognize all variables with spatial dimensions (including source size parameters) and may make use of negative SI values, address omitted variable bias or use an explicit multiple-source formulation. An alternative approach using homogeneity via differential similarity transforms is probably the best way forward. An error in the literature is corrected: the gravity SI for a finite step is  $-1$ , but it requires a more generalized formulation. We develop a new terminology, *fractional SI*,  $s$ , which is permitted to take fractional values and makes no pretense to honor concepts of homogeneity.

### INTRODUCTION

The structural index (SI) is a fundamental parameter in potential-field depth estimation. It occurs implicitly or explicitly in virtually all depth estimation techniques because it encapsulates a fundamental truth — source depth information ultimately resides in the field curvature. But, the field curvature from a source depends on the geometrical nature of the source. The SI conveniently expresses the geometrical nature of the source, thereby allowing decoupling of the depth and geometrical contributions to the observed field. The SI concept was originally developed in the context of depth estimation using Euler homogeneity (Hood, 1965; Ruddock et al., 1966; Slack et al., 1967; Thompson, 1982; Reid et al., 1990) but has also found wide use in automated methods such as source parameter imaging (SPI) (Thurston and Smith, 1997), source location using total-field homogeneity (Thurston and Smith, 2007), tilt depth (Salem et al., 2007), ADEPT (Phillips, 1979), depth from extreme points (Fedi, 2007), and curvature itself (Phillips et al., 2007). It is

embedded in methods such as those of Werner (1955), Hartman et al. (1971), and Naudy (1971), which separate dike and step cases. It is also implicit in many classic manual methods (Henderson and Zeitz, 1948; Peters, 1949; Vacquier et al., 1951; Smellie, 1956).

These and most subsequent developments were restricted to the special case of sources that have no length parameters (thickness, width, depth extent) and are thus characterized by a lone singularity beneath the profile (sphere center, line end, line, thin bed fault, sheet edge, and infinite step). All the model size parameters (apart from the irrelevant sphere and cylinder radius) are either negligible or infinite. Such “single-point” sources have one vital factor in common. Their anomalous fields may be described in terms of a simple power law. This assumption is critical to their simplicity and their wide adoption but is not a guarantee that they are universally applicable to interpretation problems in real geology, which includes less tractable structures such as thick sheets, finite steps, and yet more complex shapes.

Reid et al. (2014) discuss the appropriateness of undertaking Euler deconvolution for a given geologic setting, data preparation

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(sampling interval, grid interval, and coordinate system) and preprocessing (gradient calculation and filtering), and proper parameter choice needed to perform a valid conventional Euler deconvolution. The parameters discussed in Reid et al. (2014) are window size, solution acceptance criteria, and SI. Here, we expand upon the discussion of the SI.

### THEORY

#### Euler homogeneity for simple sources

The SI concept has its roots in Euler’s homogeneity relationship. Following Courant and John (1965), a function  $f(\mathbf{v})$  depending on variables  $\mathbf{v} = (v_1, v_2, \dots, v_i)$  is said to be homogeneous of degree  $n$  if

$$f(t\mathbf{v}) = t^n f(\mathbf{v}), \tag{1}$$

where  $\mathbf{v} = (v_1, v_2, \dots, v_i)$  is the set (vector) of an arbitrary number of variables (components) with respect to which the homogeneity of the field  $f$  is tested;  $t$  is a real number, say  $t > 1$ ; and  $n$  is the degree of homogeneity of  $f(\mathbf{v})$ . The variable  $n$  is an integer. Equation 1 is a statement about scaling behavior.

Such a homogeneous function also satisfies Euler’s differential equation:

$$\mathbf{v}\nabla f(\mathbf{v}) = n f(\mathbf{v}). \tag{2}$$

Although this formulation is much more general, it does apply to the particular case of single-point sources, which follow simple power-law field fall-off functions. Such sources show homogeneous gravity and magnetic fields, as listed in Table 1. Following convention, we define the SI as

$$N = -n. \tag{3}$$

The applicable power law takes the form of

$$F = C/r^N, \tag{4}$$

**Table 1. Structural index for magnetic (M) and gravity (G) sources of different source geometries.**

Source	Smellie model	SI (M)	SI (G) <sup>3</sup>
Sphere <sup>4</sup>	Dipole	3	2
Vertical line end (pipe)	Pole	2	1
Horizontal line (cylinder) <sup>4</sup>	Line of dipoles	2	1
Thin bed fault <sup>4</sup>	Line of dipoles	2	1
Thin sheet edge	Line of poles	1	0
Finite contact/fault <sup>5</sup>	—	0? <sup>6</sup>	-1
Infinite contact/fault	—	0	7

<sup>3</sup>Some of these elemental gravity models would be difficult to observe in practice.

<sup>4</sup>Gives depth to center, not depth to top.

<sup>5</sup>To be handled properly in a valid homogeneity framework, this model requires a more complete nonlinear formalism incorporating the thickness and recognizing multiple source edges in the same window (Stavrev and Reid, 2007, 2010) — See text.

<sup>6</sup>We are not aware of any published proof of this value, but we suggest it on grounds of its relationship to the gravity SI of a source with the same geometry.

<sup>7</sup>The infinite block/contact/fault model has an infinite gravity field anomaly. It is not a useful model.

where  $F$  is the gravity anomaly or total-field magnetic anomaly;  $C$  is a constant, which includes factors such as anomalous mass or magnetization; and  $r$  is the distance between the source critical point and the observation point. Henderson and Zeitz (1948) and Smellie (1956) propose much the same set of elemental sources, although they describe them in terms of simple assemblages of poles or dipoles (Table 1). For the cases of the sphere (point source) and the horizontal cylinder (line source), these methods give the depth to center and not the more desirable depth to top. Their radii, famously, do not affect the shape of the observed anomaly field and may therefore be neglected quite legitimately.

In practice, for the single-point case, equation 2 is presented in the form,

$$(x - x_0) \frac{\partial F}{\partial x} + (y - y_0) \frac{\partial F}{\partial y} + (z - z_0) \frac{\partial F}{\partial z} = N(B - F), \tag{5}$$

where  $B$  is the constant background value of the field  $F$ .

### DISCUSSION

#### Integer nature of the SI

It is vital to remember that  $N$  and  $n$  are integers. This is innate to the homogeneity/scaling formulation. Noninteger values represent a departure from the basic assumptions of homogeneity and single-point sources, which are necessary to obtain an easily solved linear equation. By consideration of the power-law special case (equation 4), some workers extend the formulation to noninteger values (e.g., Thompson, 1982; Reid et al., 1990; Tedla et al., 2011), but noninteger values of  $N$  or  $n$  are inevitably inconstant under variation of the source-observation vector (Courant and John, 1965; Steenland, 1968; Ravat, 1996). They have been thought to represent transitional cases between the simple power-law sources listed in Table 1. Because noninteger values of  $N$  are not constant for a given source, they violate the fundamental assumptions of the technique. Thus, a finite step viewed sufficiently close will show magnetic anomaly behavior of an infinite step, and from sufficiently far, it will be seen as thin, with all intermediate distances yielding transitional behavior.

Theoretical considerations rule out the notion of describing transitional behavior with a fractional index. Recall that a harmonic function in two variables (say,  $x$  and  $z$ ) is the real part of a complex analytic function. That is, if  $f(x, z)$  is analytic, i.e.,

$$f(x, z) = u(x, z) + iv(x, z), \tag{6}$$

then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} = 0. \tag{7}$$

Furthermore, all the singularities of a potential field are poles, and these poles, by definition, are of integer order. Thus,  $x$  and  $z$  must have only integer exponents. This implies that  $u(x, z)$  and  $v(x, z)$  are integral ordered, homogeneous, or otherwise.

The same point may be made in a more graphic or “practical” way by considering an infinite step, a thick step, and a “blob” source body (Figures 1–3).

Figure 1 shows an infinite step. It only requires one vector  $\mathbf{r}$  to connect to its edge. This case is easily treated, using a magnetic SI of zero (Reid et al., 1990). There is no equivalent gravity case because that has an infinite anomaly (Table 1).

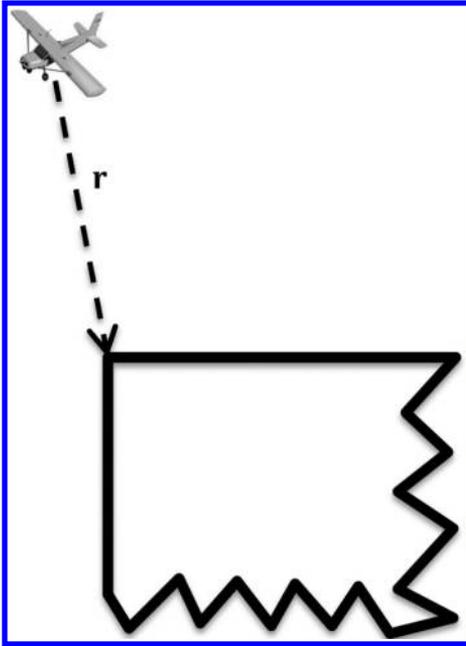


Figure 1. A block edge or infinite contact/fault (Table 1) forming a single-point source body with position defined by one ( $x$ ,  $y$ , or  $z$ ) location. We work with one distance variable  $\mathbf{r}$ . The magnetic anomaly displays simple single-point homogeneity and is easily handled by simple Euler deconvolution to locate the block edge.

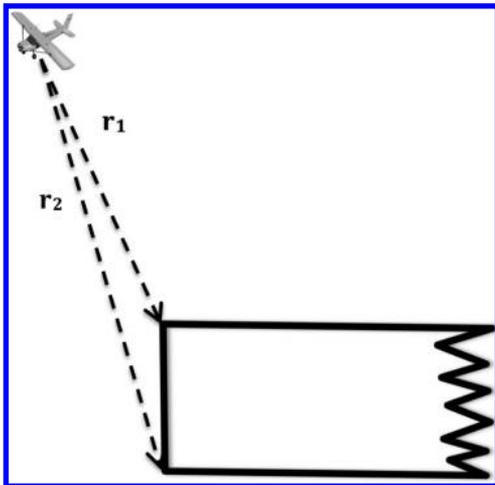


Figure 2. A thick block or finite contact/fault (Table 1) with two edges forming a multiple-point source body with position defined by two ( $x$ ,  $y$ , or  $z$ ) locations. Simple Euler deconvolution cannot cope with two  $\mathbf{r}$  values. If we neglect  $\mathbf{r}_2$  and work only with  $\mathbf{r}_1$ , we find we have a fractional SI (fSI), variable with observation location. The position tends to be indicated some way down the edge. Alternatively, we require a more sophisticated treatment.

Figure 2 shows a thick step. It will often be a good model for a fault. We now require  $\mathbf{r}_1$  and  $\mathbf{r}_2$  to locate its top and bottom edges. Its anomaly is effectively the difference between two infinite blocks such as that in Figure 1. This is not a good model for simple Euler deconvolution, although it is common and geologically interesting. If we insist on using only one vector, we probably prefer  $\mathbf{r}_1$  to the top edge, but the SI now becomes a function of  $\mathbf{r}_1$ , and for the magnetic case, it can vary from zero (close to the edge) to one (distant from the edge) at a rate that depends on the step thickness.

Figure 3 shows an irregular source body, which is always a useful source type, but generally intractable. We will often be interested in  $\mathbf{r}_T$ , the distance to the top, but many methods (including simple Euler deconvolution) will respond to  $\mathbf{r}$ , which points to some undefined location within the irregular body. The end point of  $\mathbf{r}$  is likely to vary as we change the observation point, and the SI will certainly be a function of the observation position and  $\mathbf{r}$ . Simple Euler deconvolution alone (or indeed any simple method) is unlikely to locate the body top directly.

### Euler homogeneity for other sources

The above restriction to simple sources with integer power-law fields is a theoretical necessity if we require simple and linear

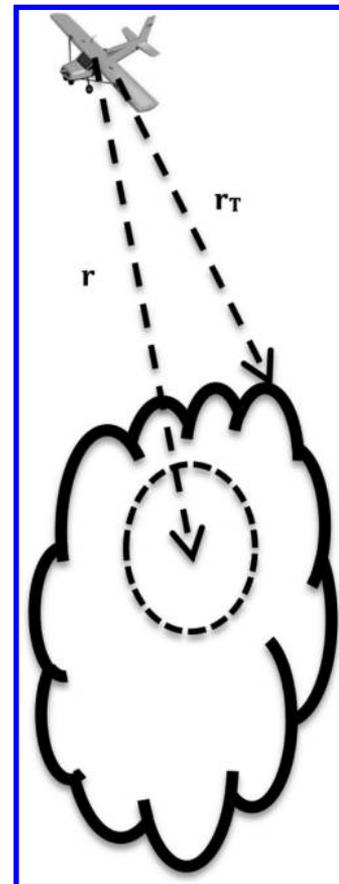


Figure 3. The general case of a source body with a complex shape. We seek the depth to top defined by the vector  $\mathbf{r}_T$ . Most geophysical systems will give us  $\mathbf{r}$ , which provides a location at some uncertain place, probably within the source body. The simple approach with one  $\mathbf{r}$  value gives a magnetic SI varying between zero and three, depending on observation position, and a body location varying between the top and the center of magnetization or center of gravity.

versions of Euler's differential equation. It is the basis of much of the work on Euler deconvolution and related methods to date. The restriction has consequences because many realistic source types such as dipping thick sheets and faults of moderate throw are impervious to the method. These sources require at least two locations along the profile (both edges) to describe them. They generate anomalous fields that cannot be written as a simple constant-coefficient power law because they are effectively the difference between two power laws (one for each edge). An enhancement to conventional Euler deconvolution overcomes this restriction by reverting to the original, unrestricted version of homogeneity as expressed by equations 1 and 2 and including all the variables with spatial dimensions (including those of the source body) in the analysis instead of assuming a sole point and referring all observation distances to that point. The formulation has been developed by Stavrev and Reid (2007) and shown in action on the gravity anomaly of a thick step in Stavrev and Reid (2010). Such a source still has a homogeneous field in the sense intended by Euler, as long as we do not impose the artificial requirement that the source body must have exactly one reference location. But the resulting Euler differential equation is not linear, and therefore the formulation is more difficult to implement (although more realistic and ultimately more rewarding).

### Solving for source depth and SI

Source depth and SI as expressed in equation 5 are not fully independent variables (Ravat, 1996; Barbosa et al., 1999; Melo et al., 2013). They are coupled strongly enough that solving for them simultaneously is an ill-posed problem. Thompson (1982) discusses a direct solution for SI but reports that his formulation returned SI and depth values that were both biased on the high side. Barbosa et al. (1999) perform a formal analysis and verify the existence and seriousness of this problem, but also develop an effective solution that exploited the value returned by solving for the background field  $B$  (equation 5). They perform the Euler analysis for multiple fixed (integer) values of SI and choose the value that minimizes the correlation between the observed field and the returned value of  $B$  for any given anomaly. The technique of extended Euler deconvolution (Mushayandevu et al., 1999, 2001; Nabighian and Hansen, 2001; FitzGerald et al., 2004) offered another approach to solving for SI and depth simultaneously. It has been implemented commercially, but in its present form it treats the SI as a continuous variable and therefore returns a range of noninteger values. We are not aware of any covariance analysis of the extended Euler formulation. We therefore cannot be sure of the reliability limits on the results when we solve for depth and SI simultaneously. The worst of the problem can sometimes be avoided by identifying and using the locations immediately above source body critical points. This is the case with the improved SPI (iSPI) (Smith et al., 1998) and analytic signal and Euler deconvolution (AN-EUL) (Salem and Ravat, 2003) methods, but the fSI so found remains even then a function of the observation height, so that it is not a source body constant.

### Incorrect SI values and some other errors in the literature

Reid et al. (1990) consider the gravity of a finite step and derive an SI value of +1. This value was subsequently used in at least one case study (Keating, 1998). Unfortunately, the derivation contained

errors and more recent work (Stavrev and Reid, 2007) using a more generalized approach has shown that the correct value is in fact  $-1$ . But this case cannot be properly addressed by most current or commercial implementations.

Reid et al. (1990) use an SI of 0.5 (among others) on a real magnetic data example. We now regard this as erroneous. Other papers, starting with Slack et al. (1967) and too numerous to mention, use noninteger SI values. As an example, we cite Tedla et al. (2011) as a particularly egregious case history. Reid et al. (2012) comment critically on this work. It uses a commercial implementation of classic grid Euler deconvolution (Reid et al., 1990) and a "depth-tuning" approach to choose an SI of 0.5 on gravity data to estimate the depth to the Moho under Africa from a satellite-derived gravity model. They choose the value of 0.5 because it gave the best average depth, but this was undermined by biases introduced by poor choices of grid interval and window size. Additionally, the value of 0.5 in the gravity case does not correspond to any plausible structure at the base of the crust, and it suffers from all the ills of the noninteger SI discussed above. The work yields demonstrably unreliable results.

In our view, it is an abuse of the method to treat the SI as a depth-tuning parameter because its chosen value has explicit meaning in terms of geologic structure.

### Alternate homogeneity approaches

The problem of interfering sources (which is what gives rise to the noninteger SI in many cases) can also be addressed by using a multiple-source approach (Hansen and Suci, 2002). This addresses the problem of thick source bodies because they can be seen as the difference between two steplike bodies. By using multiple order gradients simultaneously, thick bodies can be handled. The price is a loss of other kinds of information, such as the geometric details, but depths can be generated more reliably.

The omitted variable bias (OVB) approach (Thurston, 2010) confronts the problem directly. It recognizes that, in simple Euler approaches, relevant variables (such as body thickness) are omitted and therefore bias the result. The method goes on to estimate the bias and apply a correction, successfully in some cases.

The homogeneity property can be exploited in a radically different way by use of differential similarity transforms (Stavrev, 1997; Stavrev et al., 2009; Gerovska et al., 2010). This approach is much more general, takes full advantage of the concept of homogeneity in a fully general sense, and allows for interfering sources and multiple (integer) SI values. It may well be the best way forward but has not yet been widely implemented.

### A parting of the ways?

For some time, the concepts of degree of homogeneity  $n$  and SI  $N$  have been uneasy bedfellows. The convention has been to recognize that, for the special source geometries of Table 1, their potential field anomalies follow an inverse power law as expressed in equation 4 and  $n = -N$ . From the earliest times (Slack et al., 1967), there has been a great temptation to extend the concept of SI to fractional values. We have made the case above that the degree of homogeneity is innately an integer. But clearly, if we are prepared to ignore relevant source parameters to simplify a problem, and thereby abandon the mathematical and physical benefits of homogeneity, fractional power-law indices are present.

Some workers may feel that we should explicitly define the SI in terms of the power-law index, permit it to take fractional values, and accept that it is no longer a source geometry constant but is a function of the vector  $\mathbf{r}$  from the observation point to the chosen anchor point on the source body. We suggest that any such practice should use a distinct terminology. We propose that such a variable power-law index be called just that (the power-law index) or be termed an fSI and that the symbol  $N$  (which by convention is normally taken to be an integer) be replaced with  $s$ . We make this proposal in recognition of the powerful forces behind an abandonment of the concepts of homogeneity in favor of a practical approach, whereas we seek to preserve the integrity of methodologies that continue to honor homogeneity.

This approach would recognize the usefulness of methods such as AN-EUL (Salem and Ravat, 2003) and iSPI (Smith, 1999), which only use observations directly above source-body edges and derive the fSI as part of the process.

## CONCLUSION

The nature of the SI requires that it be an integer. Ultimately, it is a scaling exponent. Noninteger values cannot be constant for a given source, under variation of the source-observation vector. It is possible to solve for the best integer SI for an isolated source by exploiting the little-used background value  $B$  returned by most practical implementations.

The simple Euler deconvolution formulation is only suited to simple source types. To cope with more realistic sources, it is necessary to use one of several more generalized approaches. These include nonlinear versions incorporating body parameters, OVB, multiple-source formulation, or differential similarity transforms.

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