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Avoidable Euler Errors – the use and abuse of Euler deconvolution applied to potential fields*

Alan B. Reid[1,2†], Jörg Ebbing[3,4] and Susan J. Webb[5]

[1]*Reid Geophysics Ltd, Eastbourne, UK.*, [2]*School of Earth and Environment, University of Leeds, Leeds, UK,* [3]*Geological Survey of Norway (NGU), Trondheim, Norway,* [4]*Department of Petroleum Engineering and Applied Geophysics, Norwegian University of Science and Technology (NTNU), Trondheim, Norway, and* [5]*School of Geosciences, University of the Witwatersrand, Johannesburg, South Africa*

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ABSTRACT

Window-based Euler deconvolution is commonly applied to magnetic and sometimes to gravity interpretation problems. For the deconvolution to be geologically meaningful, care must be taken to choose parameters properly. The following proposed process design rules are based partly on mathematical analysis and partly on experience.

1. The interpretation problem must be expressible in terms of simple structures with integer Structural Index (SI) and appropriate to the expected geology and geophysical source.

2. The field must be sampled adequately, with no significant aliasing.

3. The grid interval must fit the data and the problem, neither meaninglessly over-gridded nor so sparsely gridded as to misrepresent relevant detail.

4. The required gradient data (measured or calculated) must be valid, with sufficiently low noise, adequate representation of necessary wavelengths and no edge-related ringing.

5. The deconvolution window size must be at least twice the original data spacing (line spacing or observed grid spacing) and more than half the desired depth of investigation.

6. The ubiquitous sprays of spurious solutions must be reduced or eliminated by judicious use of clustering and reliability criteria, or else recognized and ignored during interpretation.

7. The process should be carried out using Cartesian coordinates if the software is a Cartesian implementation of the Euler deconvolution algorithm (most accessible implementations are Cartesian).

If these rules are not adhered to, the process is likely to yield grossly misleading results. An example from southern Africa demonstrates the effects of poor parameter choices.

Key words: Gravity, Interpretation, Magnetics, Potential Fields, Euler Deconvolution.

INTRODUCTION

The interpretive technique commonly known as Euler Deconvolution was first proposed in a workable form applied to magnetic profile data by Thompson (1982). Reid *et al.* (1990)

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†E-mail: alan@reid-geophys.co.uk

implemented Thompson's suggested approach to gridded data, developed the special case for the magnetic field of a contact of infinite depth extent and coined the term "Euler Deconvolution". Since then, the method has been widely applied to magnetic data and also applied to gravity (Keating 1998), gravity vertical gradient (Klingele, Marson and Kahle 1991), and tensor gravity gradient (Zhang *et al.* 2000). It has further been extended (Mushayandebvu *et al.* 2001; Ravat *et al.* 2002) and generalized to cope with a wider range of source types (Stavrev and Reid 2007, 2010). All of these techniques employ some kind of moving data window.

The technique has been widely implemented in academic and government circles. There are at least two commercial implementations. Geologically useful results have been obtained by many workers after careful data preparation and intelligent choice of processing parameters. Conversely, poor parameter choice can yield grossly misleading results. This presentation lays out guidelines for informed data preparation and parameter choice.

EULER DECONVOLUTION THEORY

The process assumes the field is "homogeneous", that is that it obeys Euler's homogeneity (or scaling) relation,

$$f(t\mathbf{v}) = t^n f(\mathbf{v}), \tag{1}$$

and hence Euler's differential equation derived from equation (1),

$$\mathbf{v}\,\nabla f(\mathbf{v}) = nf(\mathbf{v}), \tag{2}$$

where $\mathbf{v} = (v_1, v_2, \ldots, v_k)$ is the set of components, t is a real scaling, and n is the degree of homogeneity of $f(\mathbf{v})$. The degree of homogeneity n is an integer. For the restricted case of source bodies which can be described with one location (x, y, z) and no finite length-dimensioned size parameters such as thickness or throw, potential fields follow the simple relation $f = 1/r^N$ where N (= $-n$) is a non-negative integer. N is commonly known as the Structural Index (SI). SI values for valid sources are shown in Table 1. Typical Cartesian implementations express equation (2) in the form

$$(x - x_o)\partial T/\partial x + (y - y_o)\partial T/\partial y + (z - z_o)\partial T/\partial z = N(B - T), \tag{3}$$

where (x_o, y_o, z_o) is the position of a source whose total field T is detected at (x, y, z) and B is the regional value of the field. All the variations on Euler deconvolution (references above) involve working through the data (profiles or grid) using a moving subset or "window". At each window position, a set of linear equations is solved to locate the source in plan and depth. Typical implementations assume an SI value as input or solve using several different values, and make a choice later. They also typically solve for the background value, B, of the anomalous field. Each window solution presupposes the existence of one simple source beneath the window.

Table 1 Structural Index values.

Model	Magnetic SI	Gravity SI
Point, sphere	3	2
Line, cylinder, thin bed fault	2	1
Thin sheet edge, thin sill, thin dyke	1	0
Thick sheet edge[a]	0[a]	−1[a]
Contact of infinite depth extent	0	Not useful[b]

[a]Requires the extended definition of SI as proposed by Stavrev and Reid (2007, 2010) and a non-linear deconvolution process.
[b]The gravity anomaly is infinite.

PRECONDITIONS FOR VALID RESULTS

Valid geological models

Before any deconvolution is undertaken, it is vital that thought be given to the geological problem being investigated and the method should only be applied to simple cases involving a single depth at any single (x,y) location. It is wise to remove any effects already well understood, such as regional gradients or terrain corrections. The solution at each window position is limited to dealing with the potential field effects of one isolated edge of one of the small set of permitted models defined by an integer SI (Table 1). It also assumes that the interfering effects from adjacent structures do not include appreciable curvatures or gradients and are only present (if at all) as a DC offset. Most implementations automatically solve for such an offset. In practice the technique is most effective in characterizing dykes, sills, normal faults or other sharp lateral changes in magnetization (or density). It is inapplicable to problems such as defining a deep undulating surface like the Moho. The undulations give rise to potential field effects that cannot be represented in the simplified terms assumed by the method. The effects of more than one source edge in any one window can only be handled by multi-source implementations such as that of Hansen and Suciu (2002).

The Euler method is therefore inapplicable to some valid geological investigations using geophysical data. It has many valid applications, but it is not a panacea.

Field anomaly

The field anomaly must be dominated by one structural edge at any one (x,y) location, so that a single depth solution has some meaning. Stavrev and Reid (2007, 2010) show how to solve for the top of a fault and its throw using a generalized implementation, but this involves solving non-linear equations and has not been implemented commercially.

Sampling

The measurements must sample the field well enough to characterize all the wavelengths present. If the sampling interval (e.g. flight lines) is too wide, it may not detect high amplitude field excursions of shorter wavelength. The "hit and miss" nature of such wide sampling causes shorter wavelength information to appear as spurious longer wavelengths and is known as "aliasing". Reid (1980) proposes magnetic field sampling criteria to avoid serious aliasing in both the field and in any measured or calculated gradients.

Grid interval

The grid interval should be as large as possible consistent with describing the field properly. Over-gridding or fine interpolation does not add information to the problem. It just adds run-time, and worsens the under-estimation of reliability. This problem is implicit in the formulations of Thompson (1982) and Reid *et al.* (1990) and remains implicit in all the implementations based on those formulations. It arises because simple calculations of error limits assume that all data values in a grid window are independent uncorrelated estimates with zero cross-covariance. That is never true for properly sampled, gridded data, so that uncertainties calculated using simple uncorrelated error methods are always underestimated. Over-gridding simply exacerbates the problem by seeming to provide lower estimated errors while increasing computer run-times.

Gradient validity

The Euler process requires valid gradients. There are two ways to obtain them – by measurement or calculation. The ideal is to measure them well, and of course gradients are increasingly being measured. Zhang *et al.* (2000) show how measured gravity tensor gradients may be used directly on line data in an Euler process to delineate structure. In that instance it was not even necessary to work with gridded data. The original line data sufficed. Such gravity tensor gradient data are becoming more readily available. Similarly, magnetic gradient data from a tri-axial magnetic gradient survey or a magnetic tensor gradient survey might be used. Any such use of measured gradients poses requirements on the gradient data, such as co-location, small enough zero offset and low enough noise.

Much more commonly, the gradients are calculated, using numerical methods. Although horizontal gradients may be calculated using splines or finite differences, vertical gradients normally require Fourier methods. The horizontal gradient calculations must obey conditions of low enough aliasing and low enough noise. The Fourier calculations impose additional conditions involving the much-publicized but frequently ignored requirements for data end extension, tapering, edge-matching and edge gradient matching. Commercial software often does an amazingly good job of hiding these difficulties and dealing with them unseen and effectively, but it is wise to check the gradient grids (or profiles) to be sure they are not suffering from the ringing associated with ineffective edge matching. We have seen too many examples of geologically nonsensical results arising from unthinking use of borrowed or commercial software.

The advice is therefore *"Check your gradient data, be they calculated or measured, to be sure they do represent the gradients of the primary data with sufficiently low noise and are free of artefacts"*.

Window size

The choice of physical window size is a compromise between conflicting requirements for high resolution, stable numerical solutions and appropriate depth of investigation. Since the data in any given window should only represent the effects of a single source (with all other sources represented by a "Background" offset), we gain in spatial resolution by making the physical window as small as possible. If the observed magnetic field shows effects from two well-separated depths (such as thin, shallow volcanics and a much deeper basement), it is sometimes possible to separate them by suitably chosen filtering (and desampling of the grid representing the deeper sources) and deconvolving for more than one depth in separate runs on the separated grids. In the process of matching grid interval, physical window size, filtering and depth of investigation, we generally find ourselves using windows containing between 5×5 and 10×10 grid points.

But in any event, the window size needs to be significantly greater than the real line spacing (for line data), or real grid spacing (for grid observations) if it is to have accurate curvature information at the scale of the window. So window widths should be a minimum of twice the line spacing. This suggested criterion is based on experience and plausibility rather than any precise calculation, but it seems unlikely that a window size smaller than the data interval (as defined above) will contain reliable curvature information. It may be that the window size needs to be big enough to permit a stable estimate of the background value and SI (Barbosa, Silva and Medeiros 1999), since Cooper (2012) has shown that we do not need to use a window at all if we assume values for the background and SI.

Additionally, depths greater than twice the window size are unreliable (Reid *et al.* 1990). So, for a window implementation, the "rule of thumb" for the window physical size is:

- *as small as possible, but*
- *greater than twice the measured data (line or grid) interval and*
- *greater than half the desired depth of investigation.*

Structural Index

The SI needs to be chosen carefully. Most formulations require a pre-specified SI. It is possible to solve for SI and depth simultaneously, but these parameters are strongly covariant, so direct simultaneous solution for both parameters is typically ill-posed, especially for non-integer SI, (Ravat, 1996, Barbosa *et al.* 1999). The SI for any given anomaly may be determined indirectly by seeking the SI value that yields least local perturbation of the calculated background value, B (Barbosa *et al.* 1999).

The SI is NOT a "tuning parameter". It has a simple geological meaning (table 1 above). If you use the wrong SI, you are asking the wrong question (for example "what is the depth to this dyke?" when there is a contact beneath you) and you should expect the wrong answer. An SI that is too high will yield over-estimated depths and vice versa.

Theoretically, SI should be an integer. Some commercial implementations permit the use of non-integer values, but any non-integer SI is also variable with distance from the source, thereby obviously invalidating the assumption that it is constant (Steenland 1968, Ravat 1996). This matter is discussed in much greater detail by Reid and Thurston (in review for *Geophysics*).

Selection of solutions

Nearly all implementations of the Euler deconvolution algorithm generate sprays of so-called "spurious solutions". They arise from a variety of causes including interference from adjacent sources, but are often from windows laterally distant from any source body. The spread from the latter cause are sensitive to, and diagnostic of the source dip (Kuttikul 1995). Most implementations of the Euler deconvolution algorithm include means to reduce the number of such spurious solutions. The means include elimination of solutions which are: laterally far from the window; outside the area of positive curvature in the Total Gradient Amplitude; low reliability (from the solution statistics); or not part of a cluster. A detailed discussion of the various means that have been proposed for selection of reliable solutions is beyond the intended scope of this paper, but it is essential that such spurious solutions be recognized and either eliminated or ignored during subsequent interpretive work.

Use of Cartesian coordinates

Equations (1) and (2) above are valid in any rational orthonormal coordinate system (Cartesian, spherical, cylindrical . . .), but most popular commercial and academic developments are in the Cartesian system – like equation (3). An apparent exception (Cooper 2012) uses cylindrical or spherical polar coordinates locally, but he is working with Cartesian input grids, and the final results are expressed in a Cartesian framework.

Two problems arise if data are expressed in spherical or "geographic" coordinates (Longitude, Latitude): calculation of the derivatives by simple use of Fourier transforms; and solution of equation (3) or its equivalent.

Fourier expansions arise naturally from solving potential field problems by separation of variables in a Cartesian system. The equivalent expansion for the spherical polar coordinate system is the system of spherical harmonics. Cylindrical coordinates give rise to Hankel and Bessel functions.

It follows from the above that Fourier calculations are typically invalid and misleading if applied to data expressed in "geographic" coordinates. In particular, gradients calculated by Fourier methods cannot be expected to have "sensible" values. Even if the study area is small and near the equator, where geographic coordinates are "pseudo-Cartesian" and have

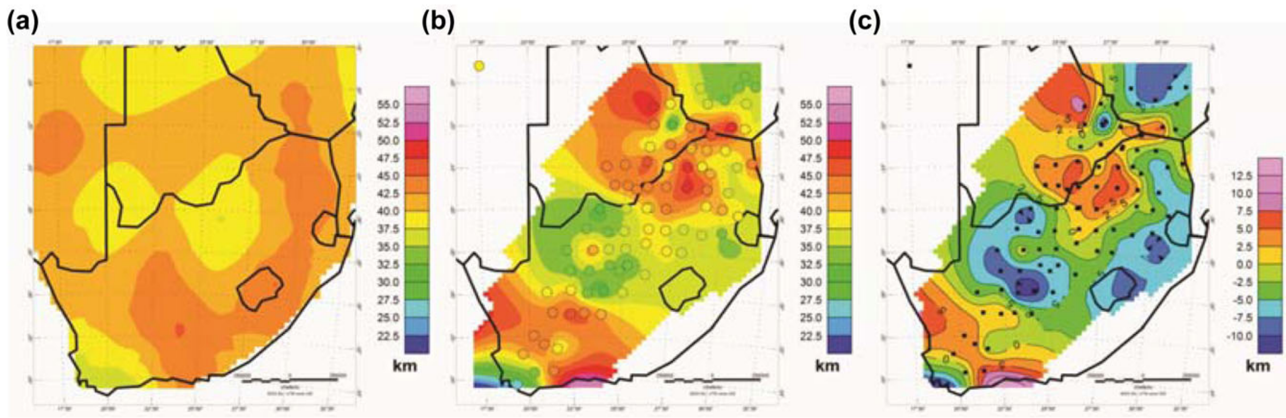

Figure 1 Southern Africa: a) Crustal thickness after Tedla *et al.* (2011), b) after Webb (2009). The circles indicate seismic stations used in compiling the thickness maps. c) Difference between the compilations in a) and b). Black squares indicate locations of seismic stations. (from Reid *et al.*, 2012, published with permission from Oxford University Press).

similar scales, Fourier-calculated gradients would very likely be expressed in nT/degree and any calculated Euler depths would be expressed in degrees (but are they degrees longitude or latitude?). Furthermore, for this case the Cartesian version of the Euler differential equation itself - equation (3) - is invalid.

A fully valid implementation of Euler deconvolution in spherical polar coordinates has been published and used successfully by Ravat *et al.* (2002). A corrected version may be found in Ravat (2011). They derived an equation equivalent to our equation (3) from the universally valid formulation of equation (2), and calculated the gradients without using Fourier transforms. Anyone wishing to work in geographic coordinates can avoid the pitfalls described above by using this implementation. But this is not a route for the mathematically naïve.

The advice is therefore simple. *"Before carrying out Fourier-based gradient calculations or performing any Euler deconvolution using conventional implementations, reproject any geographic data to a carefully chosen projection so that the process can be carried out in Cartesian space. Choose the projection to minimize distortions over the area of interest."*

REAL DATA EXAMPLE

By way of illustration, we refer to a recent paper by Tedla et al., (2011) and our own comment on it (Reid, Ebbing and Webb *et al.* 2012). This paper is an example of the misleading results that can be obtained if the guidelines above are not followed.

The original data were satellite-derived gravity values from the EIGEN-GL04C global gravity model, which is a spherical harmonic model of order and degree 360, so that only wavelengths longer than 1° (λ=110 km at the equator) are represented in the data. The data are equivalent to free air gravity. These data were interpolated and reprojected to an interval of ~ 5 km. Then Euler deconvolution was undertaken using a commercial implementation of the exact algorithm described by Reid *et al.* (1990) using a square grid window of side 20 km and an SI value of 0.5. This SI value was chosen because it yielded the best average depth over a test area, although the correlation between Euler depths and seismic depths in the test area was near-random. The resulting depths were presented as estimates of the depth to the base of the crust. Some of the results are shown in Figure 1 below, and compared with seismic depth estimates.

The results do not agree at all. We believe this discrepancy arises for five reasons.

- The input data were effectively free air gravity, so that the full topographic signal (at longer wavelengths) is present in the data, and consequent gross variation in topography will likely be represented in the estimate of the depth to base of the crust.
- The existence of any major density inhomogeneities in the crust (such as the Karoo Basin or the Karoo Volcanics) was ignored.
- The data were grossly over-gridded (5 km from 1° data).
- An Euler window size of 20 km was applied to data containing shortest wavelengths of ~200 km. Any curvatures present will be grossly under-estimated.

- An SI of 0.5 was used. This SI value applied to gravity implies the assumption of a deconvolution model that is somehow intermediate between a line source and a thin sheet edge (Table 1). The SI was explicitly chosen to give the right average depth and for no other stated reason.

The method is inapplicable to the proposed model (an undulating surface). The several errors in parameter choice can be expected to bias the depth estimates variously both low and high, while introducing very high levels of uncertainty. The over-gridding and over-simplified confidence limits provided by commercial software mask this uncertainty to a significant extent. The gross effects of the biases approximately cancel, so that the final average depth is about right, but in consequence the actual detail is unreliable.

CONCLUSIONS

The above discussion lays out the factors that must be considered if simple window-based Euler deconvolution is to yield geologically useful results. The example illustrates most clearly that inattention to the basic principles of the method can produce grossly misleading results. In summary, the recommended practice is as follows.

1. The interpretation problem must be expressible in terms of simple structures with integer Structural Index (SI) and appropriate to the expected geology and geophysical source. Consequently, for the permitted 2D source types in the cross-strike direction, source dimensions must be: vanishingly small (e.g. thin dyke);or infinitely large (e.g. sloping contact), relative to the depth. Furthermore, the source parameters (width, susceptibility, dip) must be isotropic along strike.
2. The field must be adequately sampled, with no significant aliasing.
3. The grid interval must fit the data and the problem, neither meaninglessly over-gridded nor so sparsely gridded as to misrepresent relevant detail.
4. The required gradient data (measured or calculated) must be valid, with sufficiently low noise, adequate representation of necessary wavelengths and no edge-related ringing.
5. The deconvolution window size must be at least twice the original observed data spacing (line spacing or observed grid interval) and more than half the desired depth of investigation.
6. The ubiquitous sprays of spurious solutions must be reduced or eliminated by judicious use of clustering and reliability criteria, or else recognized and ignored during interpretation.
7. The coordinate system used to express the input data should match the coordinate system used to calculate gradients and the implementation of the Euler Deconvolution algorithm. If a Cartesian implementation (e.g. any of the current commercial systems) is being used, the process should be carried out using Cartesian coordinates.

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