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Degrees of homogeneity of potential fields and structural indices of Euler deconvolution

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ABSTRACT

Homogeneity is a well-known property of the potential fields of simple point sources used in field inversion. We find that the analytical expressions of potential fields created by sources of complicated shape and constant or variable density or magnetization also show this property. This is true if all variables of length dimension are involved in the test of homogeneity. The coordinates of observation points and the source coordinates and sizes form an extended set of variables, in relation to which the field expression is homogeneous. In this case, the principal definition of homogeneity applied to a potential field can be treated as an operator of a space transform of similarity. The ratio between the transformed and original fields determines the value and sign of the degree of homogeneity n. The latter may take on positive, zero, or negative values. The degree of homogeneity depends on the type of field and on the assumed physical parameter of the field source, and can be nonunique for a given field element. We analyze the potential field of one singular point as the simplest case of homogeneity. Thus, we deduce results for the structural index, $N = -n$, in Euler deconvolution. The structural index can also be positive, zero, or negative, but it has a unique value. Analytical considerations, as well as numerical tests on the gravity contact model, confirm the proposed physical interpretation of N, and lead to an extended version of Euler's differential equation for potential fields.

INTRODUCTION

Euler's differential equation for homogeneous functions has found wide application as a theoretical basis for the inversion of large magnetic and gravity data sets in terms of simple sources (Thompson, 1982; Reid et al., 1990; Marson and Klingele, 1993; Zhang et al., 2000; Reid et al., 2003). Homogeneity can also be established and put to practical use by the direct application of its original definition.

A function is homogeneous if all of its terms have an equal sum of exponents. This follows J. Bernoulli's and Euler's concept of homogeneity (see Euler, 1936, p. 93), which was later expressed by the equivalent form

$$f(tv_1,tv_2,\ \dots\ ,tv_j) = t^n f(v_1,v_2,\ \dots\ ,v_j), \tag{1}$$

where $f(v_1,v_2,\dots,v_j)$ is a function of a set of variables $\mathbf{v} = (v_1,v_2,\dots,v_j)$, j is an arbitrary number of variables, t is a real number, and n is the degree of homogeneity of $f(\mathbf{v})$, e.g., Courant and John (1965). For such functions, if they have a differential at $\mathbf{v}$, the following Euler's partial differential equation (Euler, 1949, p. 157) holds:

$$v_1\partial f/\partial v_1 + v_2\partial f/\partial v_2 + \ \cdots\ + v_j\partial f/\partial v_j = nf(v_1,v_2,\ \dots\ ,v_j). \tag{2}$$

Conversely, if a function satisfies equation 2, then it is a homogeneous function of $v_1,v_2,\dots,v_j$ according to definition 1 (Courant and John, 1965).

(The term *homogeneous field* is also used when portions of a scalar or vector field in some part of the space are constant.)

The property of potential fields expressed by equations 1 and 2 can be referred to as *Euler homogeneity*. Thus, Euler's equation 2 is used as a condition for homogeneity in contrast to the other meanings attributed to the term *homogeneity*.

The concept of homogeneity finds application in methods for modeling physical phenomena and, in particular, for solving direct and inverse problems for potential fields. The analytical expressions used in these problems may or may not be homogeneous with respect to different sets of variables. A given expression is homogeneous if it satisfies either the defining equation 1 or equation 2 for a certain combination of variables. These conditions were tested on an analytically derived expression of gravity and magnetic field ele-

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ments for some simple sources (Stavrev, 1997). In this paper, the homogeneity test is applied to the general integral expressions for the gravity and magnetic potentials and their spatial derivatives. We consider a set of variables having the dimension of length. They are coordinates of the observation points as well as geometrical source parameters.

Equation 1 is a much easier way to analyze homogeneity than differential equation 2, which requires a lot of tedious and error-prone differential calculus. The left-hand side, $f(t\mathbf{v})$, of equation 1 can be viewed as an operator acting on the set $\mathbf{v}$ of all independent variables with dimension of length, and generates a similarity transform in the space. The ratio $f(t\mathbf{v})/f(\mathbf{v})$ determines the sign and value of the degree of homogeneity. Indeed, by equation 1,

$$n = \ln[f(t\mathbf{v})/f(\mathbf{v})]/\ln t \qquad (3)$$

when $t > 0$ and $t \neq 1$. The coefficient of similarity t can be taken to be a limited number greater than 1, without invalidating the generality of the above statements.

The multiplication of all variables of length dimension in the left-hand side of equation 1 by a given number t is a manipulation of the analytical expression that affects the observations and the sources as well. The scaling factor t is a coefficient of similarity between the original source and the transformed source. The latter copies the shape of the original source, but in another scale and position. Thus, a straight line shifts to a straight line, the angle between two lines is preserved, the length measure increases by factor t, the surface measure by t^2, and the solid measure by t^3. During the above manipulation, if we assign the original density distribution to the similar points of the transformed source, then the ratio between the original field and the transformed field will be determined by t^n, where n represents the degree of homogeneity according to equations 1 and 3. Here, we assume the mass and magnetic moment, and their line, surface, and volume densities are limited in value. It is also assumed that they are continuously (or piecewise-continuously) distributed physical parameters of limited variations in the analyzed expressions of potential fields.

With these conditions in mind, we study the geometrical and physical meaning of homogeneity shown by the general analytical expressions for gravity and magnetic fields. The studies are both analytic and numeric.

ANALYSIS OF EULER HOMOGENEITY OF GRAVITY FIELD ELEMENTS

Potential of the field of a set of point masses

The gravity field of a point mass has the potential V,

$$V = \gamma m/r, \qquad (4)$$

where γ is the gravitational constant, m is the mass, and

$$r = [(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2} \qquad (5)$$

is the distance between an observation point $P(x,y,z)$ and the location $M(x_0,y_0,z_0)$ of the mass (e.g., Blakely, 1995). In expression 4, the distance r is variable with the dimension of length. It is defined by the components $r_x = (x - x_0)$, $r_y = (y - y_0)$, and $r_z = (z - z_0)$ of the distance vector $\mathbf{r}$ from point M to point P, as well as by the elementary independent variables x, y, z, x_0, y_0, and z_0, in equation 5. The tests of homogeneity for the potential in equation 4 can be implemented using equation 1 and taking into account equation 5 to obtain

$$V(tr) = \gamma m/(tr) = t^{-1}V(r), \qquad (6)$$

$$V(tr_x,tr_y,tr_z) = \gamma m/\{[t(x - x_0)]^2 + [t(y - y_0)]^2 + [t(z - z_0)]^2\}^{1/2} = t^{-1}V(r_x,r_y,r_z), \qquad (7)$$

$$V(tx,ty,tz,tx_0,ty_0,tz_0) = \gamma m/[(tx - tx_0)^2 + (ty - ty_0)^2 + (tz - tz_0)^2]^{1/2} = t^{-1}V(x,y,z,x_0,y_0,z_0). \qquad (8)$$

These equations show that the coordinates x, y, z, x_0, y_0, z_0 form the full analytical set of independent variables of homogeneity for the gravity potential V with a degree of homogeneity $n = -1$. The minus sign implies

$$V(tx,ty,tz,tx_0,ty_0,tz_0) < V(x,y,z,x_0,y_0,z_0). \qquad (9)$$

(See equation 3 for $t > 1$). Obviously, the potential at point $P^*(tx,ty,tz)$, created by a mass m located at point $M^*(tx_0,ty_0,tz_0)$, has a lower value than the original potential at point $P(x,y,z)$. Physically, equation 9 is explained by the larger distance $tr > r$, although the mass m remains unchanged. Geometrically, the points P^* and M^* are images of similarity of the original points P and M (Figure 1).

If we accept the condition $0 < t < 1$, then inequality 9 would produce the reverse ($>$) result. However, the degree of homogeneity remains the same, namely $n = -1$. Thus, the condition $t > 1$ is sufficient for further analysis of the degree n.

A set of point masses creates a gravity field with potential,

$$V = \Sigma V_i = \gamma \Sigma m_i/r_{MiP}, \quad i = 1,2, \ldots ,j, \qquad (10)$$

where j is the number of masses, r_{MiP} is the distance between the observation point $P(x,y,z)$ and the point $M_i(x_{0i},y_{0i},z_{0i})$ locating the mass m_i. Expression 10 is homogeneous of degree $n = -1$ with respect to the set of all coordinates, $\mathbf{v} = (x,y,z,x_{01},y_{01},z_{01}\ldots,x_{0j},y_{0j},z_{0j})$, because each potential V_i is homogeneous of degree $n = -1$ by equations 6–8. Thus, all terms in equation 10 satisfy the principal requirement of an equal degree of homogeneity. The sign of n shows that $V(t\mathbf{v}) < V(\mathbf{v})$. The masses m_i, $i = 1,2,\ldots,j$, do not change their values in this homogeneity test, i.e., $m_i^*(tx_{0i},ty_{0i},tz_{0i}) = m_i(x_{0i},y_{0i},z_{0i})$.

Field potential of a line mass

Assume that a line mass is distributed continuously at a constant or variable density λ along a curved line L. Then, the potential V of its gravity field can be expressed by the well-known integral forms

$$V = \gamma \int_{(L)} dm/r_{MP} \qquad (11)$$

and

$$V = \gamma \int_{(L)} \lambda dl / r_{MP}, \tag{12}$$

where $P(x,y,z)$ is the observation point, $M(x_0,y_0,z_0)$ is a point of the line source L, and r_{MP} is the distance between points P and M according to equation 5.

The subintegral expression of the integral 11 is a homogeneous function of the distance r_{MP}, like the potential of a point mass (see equations 4–8). As a limit of the sum of an infinite number of infinitesimal terms of equal degree of homogeneity $n = -1$, the first integral has the same degree of homogeneity (see equation 10 for example). A space transform of similarity shifts the original source L to a transformed source L^* with the same mass, because this physical parameter does not change its value in the above test of homogeneity.

The second integral 12 has a subintegral expression containing two elements of length dimension, r_{MP} and dl. Their ratio, dl/r_{MP}, is a dimensionless quantity, so the degree of homogeneity of the subintegral expression is $n = 0$. Hence, the potential V obtains the same degree of homogeneity, $n = 0$. In this case, the line density $\lambda(M)$ does not change its value in a space transform of similarity, i.e., $\lambda(M^*) = \lambda(M)$, where M^* is the similar image of the original point M of the line source L.

Evidently, the potential V in expressions 11 and 12 can exhibit different degrees of homogeneity. These correspond to various ways in which the potential can be expressed whenever the mass is distributed continuously. When the mass parameter is assumed, as in expression 11, then $n = -1$. In terms of the similarity transform, this means $V^*(P^*) < V(P)$, because of the larger distances r_{M^*P} and length of L^* at the same mass of L^* as the original mass of L. When the line density is used, as in expression 12, then $n = 0$. In this case, the mass of L^* is greater than the mass of L, as the density $\lambda^* = \lambda$ on a longer line L^* (see Figure 2). This compensates for the effect of the increased distances, so that $V^*(P^*) = V(P)$. The degree $n = 0$ reflects this equality.

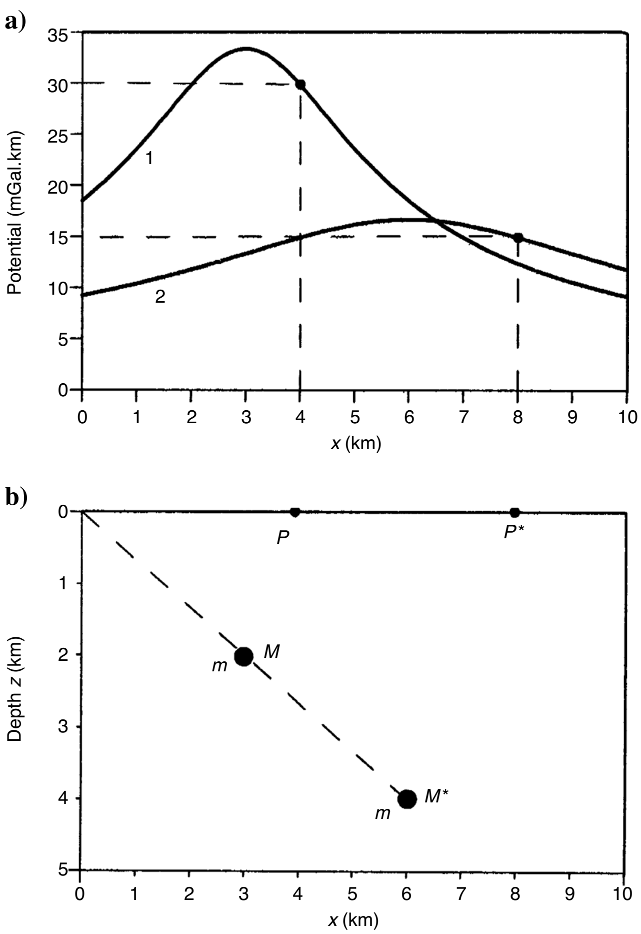

Figure 1. Similarity transforms of a point mass and the potential of its gravity field at coefficient of similarity $t = 2$. (a) Profiles of the gravity potential V (curve 1) from the original mass and the potential V^* (curve 2) from the similar mass. (b) Original mass m in point $M(3,2)$ and the similar mass $m^* = m$ in point $M^*(6,4)$. Point $P^*(0,8)$ is the similar image of the original observation point $P(0,4)$. In point P^*, the calculated gravity potential $V^* = 15$ mGal.km is half the original potential $V = 30$ mGal.km at point P, thus showing degree of homogeneity $n = -1$ ($V^* = 2^{-1}V$).

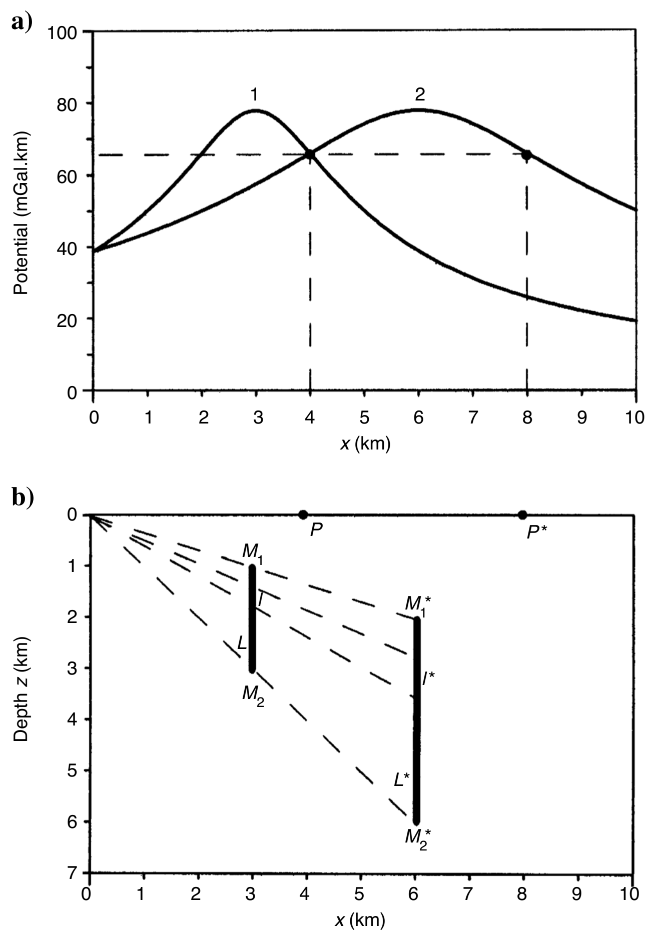

Figure 2. Similarity transforms of a vertical rod and the potential of its gravity field at coefficient of similarity $t = 2$. (a) Profiles of the potential V (curve 1) from the original rod L and the potential V^* (curve 2) from the similar rod L^*. (b) Original rod from points $M_1(3,1)$ to $M_2(3,3)$ and the similar rod from points $M_1^*(6,2)$ to $M_2^*(6,6)$ at the same density, $\lambda^* = \lambda$. A segment l^* of L^* is the similar image of the segment l of L. Point $P^*(0,8)$ corresponds to the original observation point $P(0,4)$. The calculated potential $V^*(P^*) = 66$ mGal.km is equal to the original potential $V(P)$. It means $n = 0$ ($V^* = 2^0V$).

The difference between the two possible degrees of homogeneity, $n = -1$ and $n = 0$, follows the relationship, $\lambda = dm/dl$, between the two physical parameters. This relationship contains one element of length dimension. Note that the integral expressions 11 and 12 reflect the degree of homogeneity without the need to perform the integration. This assumes the above condition requiring a well-behaved mass distribution.

The general inferences made here are confirmed by the analytical and numerical tests on the gravity potential of a line mass model with constant and variable density described in Appendix A. Appendix B shows similar basic results for a line mass with a logarithmic 2D potential.

Field potential of a surface mass

A surface mass is distributed continuously at constant or variable density σ on a surface S. Its gravitational potential is

$$V = \gamma \int_{(S)} (\sigma/r)ds \tag{13a}$$

or

$$V = \gamma \int_{(S)} dm/r. \tag{13b}$$

Homogeneity analysis can be performed by analogy with the above case of a line mass. For the first integral 13a, we obtain a degree of homogeneity of $n = -1 + 2 = 1$, taking into account the factor r^{-1} and a 2D integration area ds. For the second integral expression 13b, the factor r^{-1} gives a degree of $n = -1$. These results do not depend on the assumed functional forms of mass distribution along the surface S. The example below illustrates these conclusions.

Consider a potential V of a spherical shell with radius R, constant density σ, and mass $m = \sigma 4\pi R^2$, centered at the point $M(x_0, y_0, z_0)$,

$$V = \gamma m/r = \gamma\sigma 4\pi R^2/[(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2]^{1/2}, \tag{14}$$

where $r > R$. The degree of homogeneity of the first expression is $n = -1$ with respect to the distance r, or the set of coordinates $\mathbf{v} = (x, y, z, x_0, y_0, z_0)$, when m keeps its value. However, the expression after the second equality (=) using the full set of geometric variables, including radius R, shows a degree of homogeneity $n = 1$, as $(tR)^2/(tr) = t^1R^2/r$. In this case, $V(P^*) > V(P)$ because of the now larger mass $m^* = \sigma 4\pi(tR)^2 = t^2 m$.

Field potential of a solid mass

Assume that a solid mass is distributed continuously at a constant or variable density ρ in a domain D. The resulting potential is

$$V = \gamma \int_{(D)} (\rho/r)d\tau \tag{15a}$$

or

$$V = \gamma \int_{(D)} dm/r \tag{15b}$$

(e.g., Blakely, 1995, p. 46). The first integral 15a is homogeneous of degree $n = 2$, because the factor r^{-1} contributes a degree of -1, and the 3D volume element $d\tau$ contributes a degree of 3, so that we have $n = -1 + 3 = 2$. The second integral 15b has a degree of homogeneity $n = -1$, determined only by the factor r^{-1}.

These results can be illustrated with the example of a solid spherical body of radius R, constant density ρ, and mass $m = \rho(4/3)\pi R^3$, centered at $M(x_0, y_0, z_0)$. Outside the sphere, the gravity potential is

$$\begin{aligned} V = \gamma m/r = \gamma\rho(4/3)\pi R^3/[(x - x_{0i})^2 + (y - y_{0i})^2 \\ + (z - z_{0i})^2]^{1/2}. \end{aligned} \tag{16}$$

This expression is homogeneous of degree $n = -1$ with respect to the distance r, or to the coordinates x, y, z, x_0, y_0, z_0. Relative to the complete set of geometric variables, $v = (x, y, z, x_0, y_0, z_0, R)$, the rightmost expression in equation 16 has a degree of homogeneity $n = 2$. This positive degree shows that $V(tv) > V(v)$ because of the increased mass $m^* = \rho(4/3)\pi(tR)^3 = t^3 m$ of the enlarged sphere. In this case, the transformed source has the same density ρ as the original source, i.e., $\rho^* = \rho$.

2D potential

In Appendix B we discuss the basic 2D case of a line-mass potential. A 2D solid mass with density ρ, and a 2D surface mass with density σ, create fields with potentials

$$V = 2\gamma \int_{(\Omega)} \rho \ln(1/r)ds,$$

$$V = 2\gamma \int_{(\Lambda)} \sigma \ln(1/r)dl, \tag{17}$$

$$\text{and } V = 2\gamma \int_{(\Gamma)} \ln(1/r)d\lambda,$$

where Ω is the cross section of a solid body, Λ is an open or closed line of the cross section of a 2D material surface, and Γ is Ω or Λ, respectively. The logarithmic function has a degree of homogeneity equal to 0 (Appendix B). Thus, the first integral yields a degree of homogeneity $n = 2$, the second $n = 1$, and the third $n = 0$. These results agree with those obtained above for the cases of the 3D potentials of a solid, a surface, and a line mass, respectively, when the assumed physical parameter is the density. When the physical parameter is the partial mass of segments of equal length along the 2D source, then $n = -1$ (Appendix B). Therefore, the 2D case does not differ from the 3D case when one accounts for the entire source geometry, and not just for the source's normal cross section. In support of this conclusion, Figures 3 and 4 depict numerical tests for 2D models of thin and thick contacts, respectively.

Homogeneity of gravity potential derivatives

If the potential V is homogeneous, then so is any derivative of V (Blakely, 1995). The degree of homogeneity n_k of the derivative $\partial^k V/\partial x^\alpha \partial y^\beta \partial z^\chi$, where $\alpha + \beta + \chi = k$, decreases by the number k with respect to the degree n_0 of the potential V, i.e.,

$$n_k = n_0 - k, \qquad (18)$$

where k is the order of the derivative, $k = 0,1,2,3,\ldots$. This relation shows the increasing exponent of the reciprocal distance $1/r$ as the order k increases. The derivatives of a given order k at different values of α, β, and χ, all have equal degrees of homogeneity. The derivatives of V with respect to the geometric variables of the source also exhibit regularity of equation 18 (see also equations A-4 in Appendix A).

A generalized expression for the degree of homogeneity

The results we have obtained so far for the degrees of homogeneity of the gravity potential and its derivatives can be combined in a common expression,

$$n_g = p - k - 1, \qquad (19)$$

where k is the order of the gravity field element in terms of the derivative of the potential V, and p is an integer whose value indicates the type of physical parameter in the analytical expression of the field element. Thus, the integer p has value: $p = 0$ for the mass parameter, $p = 1$ for the line density, $p = 2$ for the surface density, and $p = 3$ for the density of a solid mass. For p between 0 and 3, the degree n_g varies between $(-k - 1)$ and $(-k + 2)$ in accordance with equation 19. Table 1 shows n_g values of the gravity field elements for orders $k \leq 3$.

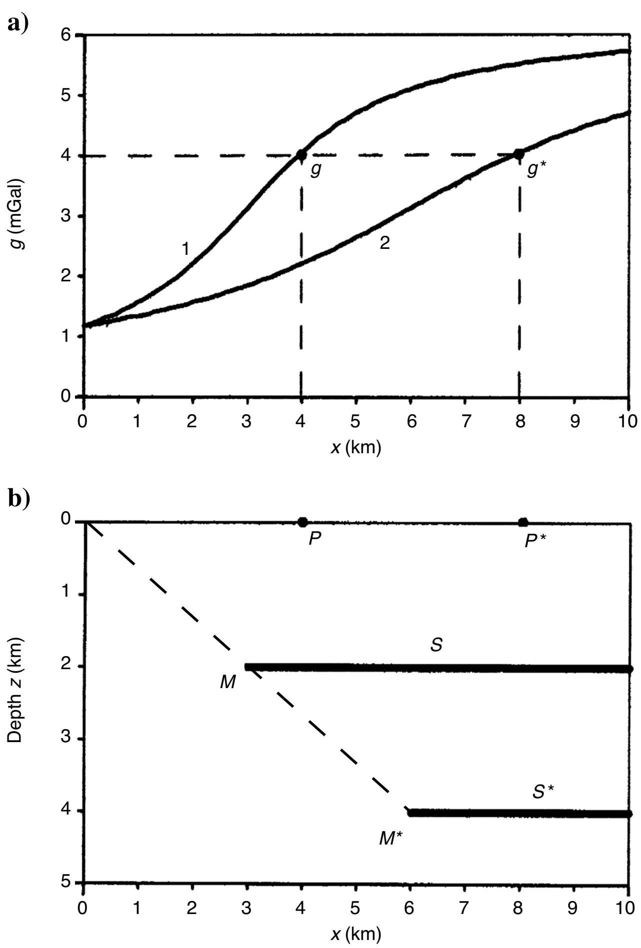

Figure 3. Similarity transforms of a semi-infinite horizontal sheet and the vertical gravity component g at coefficient of similarity $t = 2$. (a) Profiles of the component g (curve 1) from the original sheet and the component g^* (curve 2) from the similar sheet. (b) Original sheet with edge point $M(3,2)$ and the similar sheet with edge point $M^*(6,4)$ at surface density $\sigma^* = \sigma$. Point $P^*(0,8)$ is the similar image of the original observation point $P(0,4)$. In point P^*, the calculated gravity component $g^* = 4.06$ mGal is equal to the original component g at point P, thus showing degree of homogeneity $n = 0$ ($g^* = 2^0 g$).

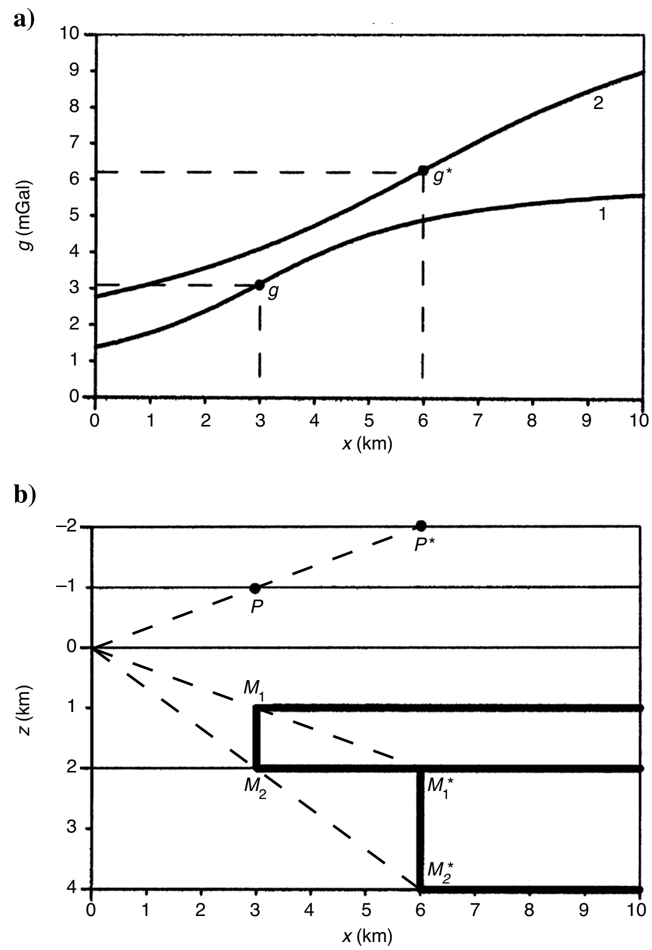

Figure 4. Similarity transforms of a vertical contact and the vertical gravity component g at coefficient of similarity $t = 2$. (a) Profiles of component g (curve 1) from the original contact and the component g^* (curve 2) from the similar contact. (b) Original contact with edge points $M_1(3,1)$ and $M_2(3,2)$, and the similar contact with edge points $M_1^*(6,2)$ and $M_2^*(6,4)$ at density $\rho^* = \rho$. Point $P^*(-2,6)$ is the similar image of the original observation point $P(-1,3)$. In point P^*, the calculated component $g^* = 6.28$ mGal is double the original component $g = 3.14$ mGal in point P, thus showing degree of homogeneity $n = 1$ ($g^* = 2^1 g$).

The parameter p in equation 19 can be treated in terms of homogeneity as the negative degree of homogeneity of the density distribution when the mass is the assumed physical parameter, i.e., when $m^* = t^0 m$. From the analysis above, $\lambda^* = t^{-1}\lambda$, $\sigma^* = t^{-2}\sigma$, $\rho^* = t^{-3}\rho$, where λ, σ, ρ are the line, surface, and solid density of the original source, and λ^*, σ^*, ρ^* are the densities of the respective transformed source. If the density is the assumed physical parameter, then the mass parameter undergoes transform, $m^* = t^p m$, in the test of homogeneity. Thus, the physical parameter distribution can be accepted as a homogeneous function with a specific conditional degree of homogeneity 0 for the assumed parameter, and p or $-p$ for the alternative parameter. R. O. Hansen (personal communication, 2005) addressed this problem.

Homogeneity of the sum of analytical expressions of a gravity field element

Consider a field element produced by collections of sources of different geometry, i.e., by point, line, surface, and solid sources. Their summary field can be homogeneous if the four potentials have the same degree of homogeneity. According to the above analysis, this is possible when the physical parameter in the appropriate analytical expressions is a mass parameter with index $p = 0$. In this case, the four potentials yield $n = -1$. The respective common degree n of a field element of higher order k is $n_g = -k - 1$, which is the minimal degree of homogeneity as indicated by equation 19.

Homogeneity as a precondition for potential field transforms

The definition of homogeneity by equation 1 can be treated as a basic expression of a field transform if all observations and geometric variables of length are involved. Equation 8 is an example of such a transform. Figures 1–5 illustrate model transforms of this type. They are space transforms of similarity with respect to a given central point O, coefficient of similarity t, and degree of homogeneity n. If a data set of gravity (or magnetic) anomaly $A(x,y,z)$ is given, then its direct similarity transform $A(tx,ty,tz)$ is given by

$$A(tx,ty,tz) = t^n A(x,y,z). \tag{20}$$

The transformed anomaly $A(tx,ty,tz)$ at shifted points $P(tx,ty,tz)$ corresponds to the shifted original source with coefficient of similarity t. Assigning n is equivalent to choosing index p (equation 19, and equation 22, below), where the order k of data A is a known number. Thus, a desired transfer of masses (or dipoles) can be implemented using the field transform by equation 20. This physical aspect of the transform can be used as a basis of inversion techniques for potential fields. Suitable tools for such inversions are the finite-difference and differential similarity transforms (Stavrev, 1997). They are functions of the differences between the transformed field and the original field at a set of common points.

ANALYSIS OF EULER HOMOGENEITY OF MAGNETIC FIELD ELEMENTS

The basic expression is the potential U of a dipole at the point $M(x_0,y_0,z_0)$,

$$U = C_m \boldsymbol{\mu} \nabla_M (1/r) = -C_m \boldsymbol{\mu} \nabla_P (1/r), \tag{21}$$

where r is the distance between point M and observation point $P(x,y,z)$, $\boldsymbol{\mu}$ is the magnetic moment, and C_m is a factor depending on the measuring units (e.g., Blakely, 1995).

The test for homogeneity with respect to the geometric variables shows that the degree of homogeneity of expression 21 is $n = -2$, because $1/r$ has a degree of homogeneity (-1), and the ∇-operator contributes (-1). The magnetic moment $\boldsymbol{\mu}$, as a vector physical parameter, is not a variable of homogeneity in this test.

As a consequence, the degree of potential U is $n = -2$, which is 1 less than the degree $n = -1$ of the potential V of a point mass (equations 6–8). The potential of more complicated dipole fixed distributions is calculated by integrating potentials $dU = C_m d\boldsymbol{\mu} \nabla_M (1/r)$ of point dipoles (equation 21). Note that the gravity potential is calculated by integrating $dV = \gamma dm/r$ of point masses (equation 15). Hence, for magnetic cases, the degree of homogeneity is always one less than the equivalent gravity case because of the ∇-operator. So, we can use equation 19 to produce an equivalent relation for magnetic cases, namely,

Table 1. Degrees of homogeneity n_g of gravity field elements.

Physical parameter in the analytical expressions of the field elements	Index, p, of the physical parameter	Gravity field elements of order k: 0 Potential	1 Strength, components	2 Gradients, curvatures	3 Third derivatives
Quantity of mass m (kg)	0	−1	−2	−3	−4
Line density λ (kg/m)	1	0	−1	−2	−3
Surface density σ (kg/m^2)	2	1	0	−1	−2
Solid density ρ (kg/m^3)	3	2	1	0	−1

$$n_m = n_g - 1 = p - k - 2, \quad (22)$$

where p is an integer specifying the type of magnetic parameter in the analytical expression, so that $p = 0$ for the magnetic moment, $p = 1$ for the line density of dipoles, $p = 2$ for the surface density of dipoles, and $p = 3$ for the volume density of dipoles (magnetization); k is the order of derivative of the potential U. This index p can be treated as a specific conditional degree of homogeneity of the magnetic parameter distributions, as in the gravity case above.

Table 2 shows the n_m values of magnetic field elements of order $k \leq 3$. For the possible values of index p, the degree of homogeneity varies within the limits $(-k-2)$ and $(-k+1)$. By analogy with the gravity case, the minimum degree, $(-k-2)$, is the common degree of homogeneity of interfering fields of point, line, surface, and solid dipole distributions. Figure 5 shows graphically the numerical test with magnetic anomaly $\Delta T(k = 1)$ of a 2.5D solid source of arbitrary polygonal cross section with constant magnetization ($p = 3$). The observed degree of homogeneity is $n_m = 0$ as predicted by formula 22.

STRUCTURAL INDICES OF EULER DECONVOLUTION

Euler deconvolution (after Reid et al., 1990) is widely used. It assumes that, at least locally, any source body giving rise to an anomalous magnetic or gravity field is simple enough to be represented by one singular point (e.g., point mass or dipole, line source, sheet edge, or thick contact top). Recently, proposals were made to deal with the field arising from a source body with two singular points (a thick layer, limited in depth contact, dike, rod, etc.) by similarity transforms (Stavrev, 1997), and for a multiple-source Euler deconvolution (Hansen and Suciu, 2002).

Suppose gravity or magnetic data are given at a set of observation points $P(x,y,z)$. The data contain the anomalous field A with one singular point $M(x_0,y_0,z_0)$. Then A satisfies equations of type 6–8 and the corresponding Euler's differential equations

$$r dA/dr = nA, \quad (23)$$

$$r_x \partial A/\partial r_x + r_y \partial A/\partial r_y + r_z \partial A/\partial r_z = nA, \quad (24)$$

$$x\partial A/\partial x + y\partial A/\partial y + z\partial A/\partial z + x_0\partial A/\partial x_0 + y_0\partial A/\partial y_0 + z_0\partial A/\partial z_0 = nA, \quad (25)$$

where n (n_g or n_m) is the degree of homogeneity of A (Tables 1 and 2). Differential equation 23 shows directly the field rate along a given radial axis r originating from the singular point M. Expression 25 is the most detailed differential equation, where $\partial A/\partial x_0 = -\partial A/\partial x$, $\partial A/\partial y_0 = -\partial A/\partial y$, $\partial A/\partial z_0 = -\partial A/\partial z$. These equalities transform equations 24 and 25 into

$$(x - x_0)\partial A/\partial x + (y - y_0)\partial A/\partial y + (z - z_0)\partial A/\partial z = nA,$$

containing only the derivatives with respect to the coordinates of the observation points. This is a case of a simplified homogeneity, compared to the more complicated cases considered above and below in this section. Field inversion for source coordinates using Euler's differential equation becomes practical because we can use calculated or measured gradients to obtain derivatives of the source coordinates.

Table 3 shows a formal conversion of the general results for the degree n in Tables 1 and 2, to the structural index, $N = -n$, of simple interpretation models. Traditionally, index N is given for the total magnetic anomalies ΔT, as it was first introduced by Thompson (1982). The other field elements obtain an index equal to N plus order k of the ΔT (or gravity Δg) derivatives. In contrast to this practice, the full value of the structural index of each field element is shown in Table 3. Thus, the values of index N can be generated directly, like the degree n in equations 19 and 22, and Tables 1 and 2. By analogy, index N in Table 3 is given by

$$N = k + s - d, \quad (26)$$

where $s = 1$ for a field element created by masses, and $s = 2$ for a field element created by dipoles. Here, $d = 0$ for a point mass, point dipole, and the equivalent spherical sources; $d = 1$ for line masses, line dipoles, and the equivalent cylindrical models; $d = 2$ for a plate

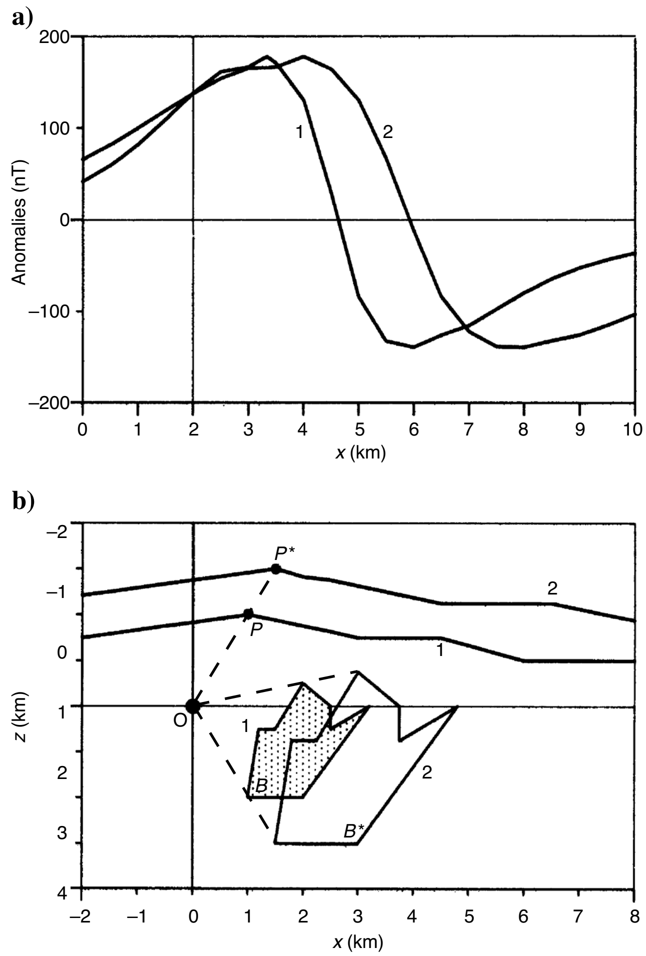

Figure 5. Similarity transforms of a complex-shaped 2.5D magnetic source and anomaly ΔT along an uneven line of observation at coefficient of similarity $t = 1.5$. (a) Profiles of anomaly ΔT (curve 1) from the original source and from the similar source (curve 2). (b) Original source B (contour 1) and the similar source B^* (contour 2) at the same magnetization vector, $\mathbf{J}^* = \mathbf{J}$. Point $P^*(1.5,-3)$ is the similar image of the original observation point $P(1,-2)$. In point P^*, the calculated magnetic anomaly $\Delta T^* = 167$ nT is equal to the original anomaly ΔT in point P, thus showing degree of homogeneity $n = 0$ ($\Delta T^* = 2^0 \Delta T$).

and the equivalent thin layer, dike, and step; and $d = 3$ for the contact model. Index d corresponds to the geometrical dimension of the elementary sources. The equivalent sources above have the same index d because their sizes (radius or thickness) are coefficients in the analytical expressions and cannot be determined uniquely in an inversion of the field. The deductive result in equation 26 gives the same values of index N as that known from the particular analysis of point source models. According to equation 26, the structural index N has unique values in contrast to the degree of homogeneity n, which depends on the type p of the accepted physical parameter.

Table 3 consists of positive, zero, and negative values of N. The latter in all cases correspond to the theoretical models with indeterminate (∞) potential and intensity. For them, $d - k > 1$ in the gravity case, and $d - k > 2$ in the magnetic case (see formula 26). The zero value of N appears for some theoretical models with indeterminate or determinate field elements, where $d - k = 1$ in the gravity case, and $d - k = 2$ in the magnetic case. For example, the vertical component of gravity intensity ($k = 1$) of a thin, semi-infinite horizontal sheet ($d = 2$) is determinate with index $N = 0$, although the sloping or vertical sheet does not have determinate vertical intensity. However, all varieties of the same magnetic model have determinate potential and intensity (see Table 3). The most specific in this respect is the gravity contact model of infinite depth extent that shows the lower formal index $N = -2$.

The physical sense of the positive N is well known from Thompson's (1982) functional form $f(x,y,z) = G/r^N$, where r is the distance in equation 5, and G is not dependent on x, y, z. If $N > 0$, then f shows a natural attenuation with distance r. If $N = 0$, then $f = G$, which is possible for some models such as the gravity semi-infinite horizontal sheet (small step). But the case $N < 0$ cannot be explained using Thompson's form. The negative structural indices in Table 3 may gain meaning in terms of the concept of extended homogeneity. The direct use of equations 1 and 3 is not possible for this purpose because the theoretical field elements are indeterminate and cannot be compared. However, an infinitesimal approach from a determinate model to the respective indeterminate one gives a reasonable answer to the problem. Such a mathematical approach corresponds to the reality where measured potential-field elements are fully determinate, so their sources may only approximate the theoretical indeterminate models. The synthetic tests shown below confirm the appearance of negative structural index N for the model of gravity contact.

Table 3 shows both well-known and little-known (but useful) regularities. The structural index N has equal value for all field elements of equal order k caused by sources of equal index d. Such elements are the magnetic field components X, Y, Z, H, magnitude T, and anomalies $\Delta T (k = 1)$, and all members of the gravity or magnetic gradient tensors ($k = 2$), also Hilbert transforms (the latter shown by Nabighian and Hansen, 2001). In all cases, N is higher by 1 for a magnetic model than N for the same gravity model.

Table 2. Degrees of homogeneity n_m of magnetic field elements.

Physical parameter in the analytical expressions of the field elements	Index p of the physical parameter	Magnetic field elements of order k: 0 Potential	1 Strength, components	2 Gradients	3 Third derivatives
Dipole moment μ (Am^2)	0	−2	−3	−4	−5
Line density β (Am)	1	−1	−2	−3	−4
Surface density η (A)	2	0	−1	−2	−3
Solid density J (A/m)	3	1	0	−1	−2

Table 3. Structural index N in Euler's differential equation for gravity/magnetic field elements.

Gravity/magnetic theoretical models	Geometrical dimension d of the model	Gravity/magnetic elements of order k: 0 Potential	1 Strength, components	2 Gradients, tensors	3 Third derivatives
Point mass/dipole equivalent sphere	0	1/2	2/3	3/4	4/5
Line of mass/dipole, equivalent cylinder	1	0′/1	1/2	2/3	3/4
Thin semi-infinite sheet, dike, sill, contact, step	2	−1″/0	0′/1	1/2	2/3
Contact of infinite depth extent, pyramid	3	−2″/−1″	−1″/0	0′/1	1/2

Note: For some model varieties (′) or for all model varieties (″) the theoretical field elements are not determinate, and N corresponds to the closest determinate model of a considerable extent.

Synthetic tests for negative indices N

Consider the gravity model of a vertical contact (Figure 4b), creating vertical gravity attraction g defined by the analytical expression

$$g = \gamma\rho\{\pi(z_2 - z_1) + 2(z_2 - z)\tan^{-1}[(x - x_0)/(z_2 - z)] - 2(z_1 - z)\tan^{-1}[(x - x_0)/(z_1 - z)] + (x - x_0) \times \ln\{[(x - x_0)^2 + (z_2 - z)^2]/[(x - x_0)^2 + (z_1 - z)^2]\}\}, \quad (27)$$

where x, z are the coordinates of observation points, and x_0, z_1 and x_0, z_2 are the coordinates of the upper M_1 and the lower M_2 edge points of the contact (e.g., Telford et al., 1990). The degree of homogeneity of anomaly g with respect to all geometric variables (x, z, x_0, z_1, z_2) is $n = 1$ according to the definition by equation 1 (see Table 1). If $z_2 \to \infty$ (infinite thickness), then $g \to \infty$ (indeterminate value). Table 3 represents this case by negative $N = -1$.

Differential equation 2 for anomaly g with respect to all geometric variables is

$$x\partial g/\partial x + z\partial g/\partial z + x_0\partial g/\partial x_0 + z_1\partial g/\partial z_1 + z_2\partial g/\partial z_2 = ng, \quad (28)$$

where $\partial g/\partial x_0 = -\partial g/\partial x$, and $\partial g/\partial z_1 + \partial g/\partial z_2 = -\partial g/\partial z$. When $z_2 \gg z_1$ and $|x| \ll z_2$, the terms $z_1\partial g/\partial z_1$ and $z_2\partial g/\partial z_2$, which contain the unknown derivatives $\partial g/\partial z_1$ and $\partial g/\partial z_2$, can be approximated by the equations

$$z_1\partial g/\partial z_1 = -z_1\partial g/\partial z - [\pi\gamma\rho z_1 + 2\gamma\rho(z_1/z_2)(x - x_0)]; \quad (29)$$

$$z_2\partial g/\partial z_2 = [\pi\gamma\rho z_2 + 2\gamma\rho(x - x_0)]. \quad (30)$$

By a substitution of these expressions in equation 28 and after simple manipulations, Euler's equation takes the form

$$x_0\partial g/\partial x + z_1\partial g/\partial z = N_2 g + x\partial g/\partial x + z\partial g/\partial z + [\pi\gamma\rho(z_2 - z_1) + 2\gamma\rho(x - x_0)], \quad (31)$$

where N_2 is an approximation to the negative structural index $N = -n = -1$. The bracketed term $[.]$ must be taken into account when solving the inverse problem of determining the upper edge point $M_1(x_0, z_1)$. This term contains unknown variables but can be included by using its linear character.

If one accepts the contact model of considerable depth extent as a one-point source with singular point $M_1(x_0, z_1)$, then Euler's equation 2 should be written as

$$x_0\partial g/\partial x + z_1\partial g/\partial z = N_1 g + x\partial g/\partial x + z\partial g/\partial z, \quad (32)$$

where N_1 is the supposed structural index. The value of this index can be estimated analytically from equation 32 by the expression

$$N_1 = [(x_0 - x)\partial g/\partial x + (z_1 - z)\partial g/\partial z]/g. \quad (33)$$

A simple test at $(x_0 - x) = 0$ when $\partial g/\partial z = 0$ and $g = \pi\gamma\rho(z_2 - z_1)$ shows $N_1 = 0$. But this is the structural index of a semi-infinite horizontal sheet model (Table 3). Hence, equation 32 does not correspond to the contact model of considerable depth extent.

A similar test can be carried out for the index N_2 in equation 31. Taking into account equations 31–33,

$$N_2 = N_1 - \pi\gamma\rho(z_2 - z_1)/g - 2\gamma\rho(x - x_0)/g. \quad (34)$$

At $(x_0 - x) = 0$ when $g = \pi\gamma\rho(z_2 - z_1)$, we obtain $N_2 = -1$ as is predicted in Table 3 for the solid contact model.

Equations 31 and 32 correspond to the models of very thick and very thin contact structures, respectively. If the real structure has an intermediate relative thickness $w = (z_2 - z_1)/(z_1 - z)$, then N_1 and N_2 values in these equations deviate from the expected ones. Calculated results from equations 33 and 34 along model profiles g above contacts are shown in Figures 6a and 7a, respectively. At a given small thickness $w = 0.02$, index N_1 maintains expected values close to 0. For a thick contact with $w = 4$, index N_1 changes significantly between -0.125 and 0.5 along the tested profile. Index N_2 (Figure 7a) shows similar properties. At a considerable thickness, $w = 19$, N_2 is close to the expected negative index -1. At an intermediate thickness, $w = 4$, index N_2 undergoes considerable changes between -1.2 and -0.1. These changes indicate a deviation of the real source from the accepted model.

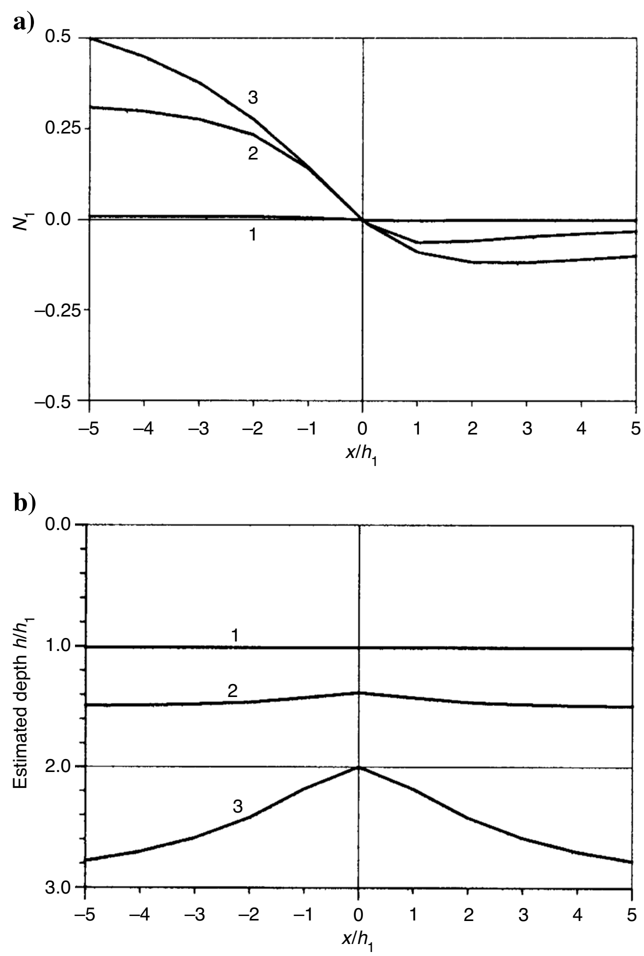

Figure 6. Structural index in Euler's differential equation for the gravity anomaly g, created by a vertical contact model, considered as a one-point source with the upper edge at depth $h_1 = (z_1 - z)$. (a) Calculated index N_1 at a relative thickness w/h_1 of the contact structure: (1) $w/h_1 = 0.02$ (thin sheet), (2) $w/h_1 = 1.0$, (3) $w/h_1 = 4.0$ (thick contact). (b) Estimated relative depth, h/h_1, from Euler's differential equation at a prescribed index $N_1 = 0$ for a relative thickness of the contact w/h_1, as in the case (a) above.

Figures 6b and 7b show calculated depths from equations 31 and 32 at prescribed indices $N_1 = 0$ and $N_2 = -1$, respectively. For an intermediate thickness of the contact model, the estimated depth may vary significantly along the interpreted profile. Using equation 32, the estimated depth tends to the average depth $(z_1 + z_2)/2$ with distance from the point above the edge (Figure 6b). For the negative index $N_2 = -1$, equation 31 gives better results near the above-edge point (Figure 7b).

Experimental results of Reid et al. (2003) on model and real gravity data find an explanation based on the above analysis. The authors obtain returned structural indices (SI) between −0.05 and 0.8, mean value of 0.5, and standard deviation of ±0.18, for a contact model. The gravity data from Penola Trough, Otway Basin in Australia, show SI values between −0.5 and 0.6, mean value of 0.11, and standard deviation of ±0.23. These results are consistent with the expected variations in the sign and amplitudes of index N_1 (Figure 6a) for intermediate contact models in equation 33.

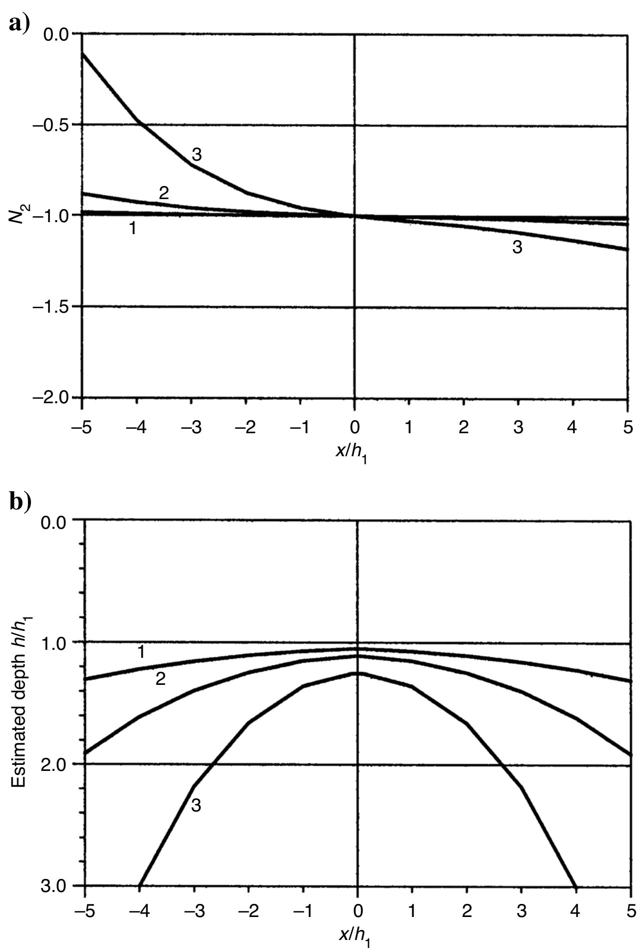

Figure 7. Structural index in Euler's differential equation for the gravity anomaly g created by a vertical contact model considered as a two-point source with the upper edge at depth $h_1 = (z_1 - z)$. (a) Calculated index N_2 at a relative thickness w/h_1 of the contact: (1) $w/h_1 = 19.0$ (considerable thickness), (2) $w/h_1 = 9.0$, (3) $w/h_1 = 4.0$. (b) Estimated relative depth h/h_1 from Euler's differential equation at a prescribed index $N_2 = -1$ for a relative thickness w/h_1, as in the case (a) above.

CONCLUSIONS

We have shown that the analytical expressions of potential field elements caused by arbitrarily distributed sources satisfy the tests of homogeneity with respect to the set of all independent length variables. The homogeneity analysis using the initial definition of homogeneity is easier than that using Euler's differential equation. A direct use of the initial definition assumes a space similarity transform involving both the observations and the sources. In terms of the similarity transform, the value and sign of the degree of homogeneity can be explained by the ratio between the transformed field element and the original field element at similar observation points. The degree of homogeneity depends on the order of the field element as a derivative of the gravity or magnetic potential and on the assumed type of the physical parameter in the analytical expression. The latter assumption leads to different degrees of homogeneity of a given field element. The existing regularities of the degrees of homogeneity can be synthesized in common formulas and shown numerically in table form for the gravity and the magnetic field elements.

The proposed extended treatment of potential-field homogeneity makes it possible to deduce results concerning theoretical and practical problems in Euler deconvolution. The structural indices reflect the regularities of the degrees of homogeneity. It is shown analytically and numerically that a negative structural index is possible for the gravity contact interpretation models of considerable depth extent. An appropriate Euler differential equation is composed.

The space similarity transform of a potential field element expresses the property of its homogeneity. This can be used to create inversion techniques for potential fields exploiting such transforms.

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APPENDIX A

HOMOGENEITY OF THE GRAVITY POTENTIAL OF A ROD WITH CONSTANT OR VARIABLE DENSITY

Consider a vertical line source (rod) L of constant line density λ, horizontal coordinates x_0, y_0, vertical coordinates z_0 between z_1 and z_2, and length $l = z_2 - z_1$. From equation 12, the potential V at the observation point $P(x, y, z)$ is

$$V = \gamma \int_{(L)} (\lambda/r) dz_0 = \gamma\lambda \ln[(z_2 - z + r_2)/(z_1 - z + r_1)], \tag{A-1}$$

where $r_i = [(x - x_0)^2 + (y - y_0)^2 + (z - z_i)^2]^{1/2}$, $i = 1,2$, are the distances from point P to the upper and lower points of the rod, respectively. The degree of homogeneity of expression A-1 in relation to the coordinates and distances is $n = 0$. The same result for n can be found from Euler's differential equation,

$$x\partial V/\partial x + y\partial V/\partial y + z\partial V/\partial z + x_0\partial V/\partial x_0 + y_0\partial V/\partial y_0 + z_1\partial V/\partial z_1 + z_2\partial V/\partial z_2 = nV = 0, \qquad \text{(A-2)}$$

after substituting the terms in the left-hand side of A-2 using expressions A-4 in the case of constant density λ.

Homogeneity is also observed when the line mass has variable density. Indeed, if $\lambda(z_0) = \lambda_1 + (\Delta\lambda/l)(z_0 - z_1)$ changes continuously from λ_1 at the upper point to $\lambda_2 = \lambda_1 + \Delta\lambda$ at the lower point of the rod, then

$$\begin{aligned} V &= \gamma\int_{(L)} \{[\lambda_1 + (\Delta\lambda/l)(z_0 - z_1)]/r\}dz_0 \\ &= \gamma\lambda_1\ln[(z_2 - z + r_2)/(z_1 - z + r_1)] \\ &\quad - \gamma(\Delta\lambda/l)\{(z_1 - z)\ln[(z_2 - z + r_2)/(z_1 - z + r_1)] \\ &\quad + r_2 - r_1\} = V_c + V_\delta, \end{aligned} \qquad \text{(A-3)}$$

where V_c corresponds to the case A-1, and V_δ is the potential caused by the density changes. The degree of homogeneity of expression A-3 with respect to all geometric variables x, y, z, x_0, y_0, z_1, z_2, is $n = 0$. The set of geometric variables is the same as in expression A-1, so Euler's differential equation will consist of the same members as in equation A-2. The derivatives of potential $V = V_c + V_\delta$ yield the following expressions

$$\begin{aligned} \partial V/\partial x &= \gamma(x - x_0)\lambda_1 E + \gamma(x - x_0)(\Delta\lambda/l) \\ &\quad \times[(z - z_1)E + 1/r_2 - 1/r_1], \\ \partial V/\partial y &= \gamma(y - y_0)\lambda_1 E + \gamma(y - y_0)(\Delta\lambda/l) \\ &\quad \times[(z - z_1)E + 1/r_2 - 1/r_1], \\ \partial V/\partial z &= \gamma\lambda_1(1/r_1 - 1/r_2) + \gamma(\Delta\lambda/l)[\ln(Q_2/Q_1) - l/r_2], \\ \partial V/\partial x_0 &= -\partial V/\partial x, \quad \partial V/\partial y_0 = -\partial V/\partial y, \\ \partial V/\partial z_1 &= -\gamma\lambda_1/r_1 + \gamma(\Delta\lambda/l) \\ &\quad \times[\ln(Q_2/Q_1)(z - z_1)/l + (r_2 - r_1)/l)], \\ \partial V/\partial z_2 &= \gamma\lambda_1/r_2 + \gamma(\Delta\lambda/l)[-\ln(Q_2/Q_1)(z - z_1)/l - (r_2 \\ &\quad - r_1)/l + l/r_2], \end{aligned} \qquad \text{(A-4)}$$

where $Q_1 = r_1 + z_1 - z$, $Q_2 = r_2 + z_2 - z$, $E = 1/(Q_2r_2) - 1/(Q_1r_1)$, and $\ell = z_2 - z_1$. An analytical proof of Euler's equation A-2 with derivatives A-4 requires a complex manipulation of their long expressions. Alternately, we can prove the result using rod models. Results from such calculations are shown in Table A-1. The observation points lie along the epicentral line on an uneven surface. As can be seen, the sum Σ of the left-hand side terms in Euler's equation A-2 is equal to 0 for all observation points. This shows degree $n = 0$, which is the same as obtained above from a direct homogeneity analysis of equation A-3.

The whole rod mass is $m = m_1 + \Delta m$, where $m_1 = \lambda_1 l$ and $\Delta m = \Delta\lambda l/2$. By substituting $\lambda_1 = m_1/l$ and $\Delta\lambda = 2\Delta m/l$ in equation A-3,

$$V = \gamma(m_1/l)\ln(Q_2/Q_1) + 2\gamma(\Delta m/l^2) \times[(z_1 - z)\ln(Q_2/Q_1) + r_2 - r_1]. \qquad \text{(A-5)}$$

Expression A-5 shows degree of homogeneity $n = -1$ with respect to the set of geometric variables z, z_1, r_1, r_2, and l. This result is identical to that obtained from the analysis of integral expression 11, when the physical parameter is the mass.

Table A-2 shows results from a numerical test with the same rod but with a variable density according to the sine function

$$\lambda(z_0) = \lambda_1\{1 + \sin[2\pi k(z_2 - z_0)/(z_2 - z_1) + \varphi]\}, \qquad \text{(A-6)}$$

where k is the frequency, and φ is the phase. Euler's differential equation A-2 is satisfied for the sinusoidal density of equation A-6 at degree of homogeneity $n = 0$. A more complicated continuous or piecewise-continuous density distribution can be represented by a Fourier series of sinusoidal components. Thus, for an arbitrary density distribution (see Introduction), Euler homogeneity of a created field element is valid.

APPENDIX B

HOMOGENEITY OF THE LOGARITHMIC 2D POTENTIAL OF A LINE SOURCE

The logarithmic potential (e.g., Blakely, 1995) introduced in the theory of 2D fields is represented here by the equivalent expression

Table A-1. The terms of Euler's differential equation A-2 and their sum Σ for the potential V of a vertical rod with a linear variation of density λ.

Coordinates of observations (km)			Potential (mGal.km)	Terms of the Euler's differential equation (mGal.km)					Sum (mGal.km)
x	y	z	V	$(x - x_0)\,\partial V/\partial x$	$(y - y_0)\,\partial V/\partial y$	$z\,\partial V/\partial z$	$z_1\,\partial V/\partial z_1$	$z_2\,\partial V/\partial z_2$	Σ
0.0	0.0	−0.1	0.368	−0.072	−0.287	−0.001	−0.037	0.397	0.000
1.0	2.0	−0.3	0.693	−0.120	−0.480	−0.030	−0.074	0.704	0.000
2.0	4.0	−0.2	2.561	0.000	0.000	−0.933	−0.607	1.540	0.000
3.0	6.0	0.1	0.725	−0.137	−0.550	0.006	−0.075	0.756	0.000
4.0	8.0	0.0	0.369	−0.072	−0.290	0.000	−0.037	0.399	0.000

Note: Rod parameters are $x_0 = 2.0$ km, $y_0 = 4.0$ km, $z_1 = 0.1$ km, $z_2 = 1.1$ km, $\lambda_1 = 0.3$ Mt/km, $\Delta\lambda = -0.1$ Mt/km.

Table A-2. The terms of Euler's differential equation A-2 and their sum Σ for the potential *V* of a vertical rod with a sinusoidal variation A-6 of density λ.

Coordinates of observations (km)			Potential (mGal.km)	Terms of the Euler's differential equation (mGal.km)					Sum (mGal.km)
x	y	z	V	$(x-x_0)\,\partial V/\partial x$	$(y-y_0)\,\partial V/\partial y$	$z\,\partial V/\partial z$	$z_1\,\partial V/\partial z_1$	$z_2\,\partial V/\partial z_2$	Σ
0.0	0.0	−0.1	0.723	−0.141	−0.563	−0.002	−0.073	0.779	0.000
1.0	2.0	−0.3	1.354	−0.232	−0.930	−0.061	−0.145	1.368	0.000
2.0	4.0	−0.2	4.670	0.000	0.000	−1.547	−1.006	2.553	0.000
3.0	6.0	0.1	1.422	−0.268	−1.073	0.013	−0.147	1.475	0.000
4.0	8.0	0.0	0.725	−0.142	−0.568	0.000	−0.073	0.783	0.000

Note: Rod parameters are $x_0 = 2.0$ km, $y_0 = 4.0$ km, $z_1 = 0.1$ km, $z_2 = 1.1$ km, $\lambda_1 = 0.3$ Mt/km, $k = 0.5$, $\varphi = 0$.

$$V = -\,2\gamma\lambda \ln(r/r_0), \tag{B-1}$$

where r_0 is a standard unit of the distance r between the line source and the observation point. This expression uses the well-known Fourier rule stating that in physical equations the transcendental functions may only have a dimensionless argument. It is easy to check that the derivatives of V do not contain the measurement unit r_0. For example, from expression B-1,

$$\partial V/\partial r = -\,2\gamma\lambda(r_0/r)(1/r_0) = -\,2\gamma\lambda/r. \tag{B-2}$$

The degree of homogeneity of potential of equation B-1 is equal to the degree of the logarithmic function with respect to r and r_0. The result is $n = 0$ as predicted by the general integral expression 12 when the physical parameter is density λ.

The density $\lambda = \Delta m/\Delta y$, where Δm is the mass of a unit segment Δy along the line source. In expression B-1, if λ is substituted for $\Delta m/\Delta y$, then the degree of homogeneity in relation to all geometric variables will change to $n = -1$. This result corresponds with the conclusion drawn for expression 11 for a mass source.

REFERENCES

Blakely, R. J., 1995, Potential theory in gravity and magnetic applications: Cambridge University Press.

Courant, R., and F. John, 1965, Introduction to calculus and analysis: Wiley Interscience.

Euler, L., 1936, Introduction to the analysis of infinitesimal: ONTI (Russian translation).

———, 1949, Differential calculus: Gostechizdat (Russian translation).

Hansen, R. O., and L. Suciu, 2002, Multiple-source Euler deconvolution: Geophysics, **67**, 525–535.

Marson, I., and E. E. Klingele, 1993, Advantages of using the vertical gradient of gravity for 3-D interpretation: Geophysics, **58**, 1588–1595.

Nabighian, M. N., and R. O. Hansen, 2001, Unification of Euler and Werner deconvolution in three dimensions via the generalized Hilbert transform: Geophysics, **66**, 1805–1810.

Reid, A. B., J. M. Allsop, H. Granser, A. J. Millet, and I. W. Somerton, 1990, Magnetic interpretation in three dimensions using Euler deconvolution: Geophysics, **55**, 80–91.

Reid, A. B., D. FitzGerald, and P. McInerny, 2003, Euler deconvolution of gravity data: 73rd Annual International Meeting, SEG, Expanded Abstracts, 580–583.

Stavrev, P., 1997, Euler deconvolution using differential similarity transformations of gravity or magnetic anomalies: Geophysical Prospecting, **45**, 207–246.

Telford, W. M., L. P. Geldart, and R. E. Sheriff, 1990, Applied Geophysics, 2nd ed.: Cambridge University Press.

Thompson, D. T., 1982, EULDPH — A new technique for making computer assisted depth estimates from magnetic data: Geophysics, **47**, 31–37.

Zhang, Ch., M. F. Mushayandebvu, A. B. Reid, J. R. Fairhead, and M. E. Odegard, 2000, Euler deconvolution of gravity tensor gradient data: Geophysics, **65**, 512–520.